

Mechanics I - Question Bank -

2018-2019 Fall semester

KEY

Full name:

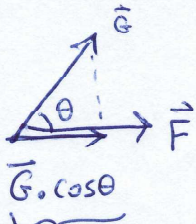
Q1. Three vectors F , v and B are related through $F = 5.0 (v \times B)$. If vector $v = 3.0 \hat{i} - 5.0 \hat{j}$ and $B = -2.0 \hat{k}$, then vector F is:? (Ans: $50 \hat{i} + 30 \hat{j}$)

$$v \times B = (3\hat{i} - 5\hat{j}) \times (-2\hat{k}) = -6(-\hat{j}) + 10\hat{i} = +6\hat{j} + 10\hat{i}$$

$$v \times B = 10\hat{i} + 6\hat{j}$$

$$5(v \times B) = \underline{\underline{50\hat{i} + 30\hat{j}}}$$

Q2. Vectors F and G are defined as $F = 3.0 \hat{i} + 4.0 \hat{j}$, and $G = -\hat{i} + \hat{j}$. Find the component (projection) of vector G along the direction of vector F . (Ans: 0.20)



This is projection of G along the direction of vector F .

$$\vec{F} \cdot \vec{G} = |\vec{F}| \cdot |\vec{G}| \cdot \cos \theta \quad |\vec{G}| \cdot \cos \theta = \frac{\vec{F} \cdot \vec{G}}{|\vec{F}|}$$

projection of G ,
on the vector F ,

$$|\vec{G}| \cdot \cos \theta = \frac{(3\hat{i} + 4\hat{j}) \cdot (-\hat{i} + \hat{j})}{|\vec{F}| = \sqrt{3^2 + 4^2}}$$

$$|\vec{G}| \cdot \cos \theta = \frac{1}{5} = \underline{\underline{0.20}}$$

Q3. The angle between the two vectors $A = 2 \hat{i} + 4 \hat{j}$ and $B = 4 \hat{i} - 2 \hat{j}$ is:? (Ans: 90 degrees)

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos \theta$$

$$(2\hat{i} + 4\hat{j}) \cdot (4\hat{i} - 2\hat{j}) = |\vec{A}| |\vec{B}| \cos \theta$$

$$8 - 8 = |\vec{A}| |\vec{B}| \cos \theta$$

$$0 = |\vec{A}| |\vec{B}| \cos \theta \rightarrow \text{This means that } \cos \theta = 0 \text{ then } \theta = 90^\circ.$$

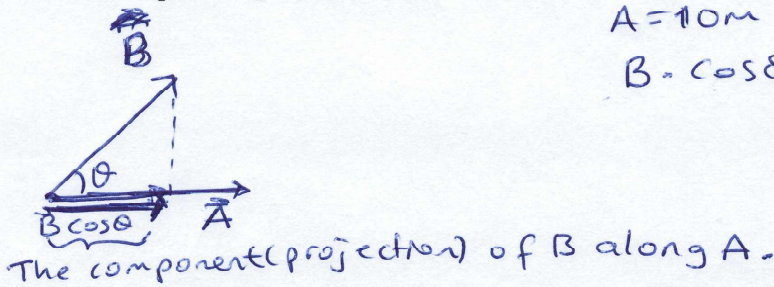
Q4. Three vectors are given as: $A = -3.0 \hat{i}$; $B = -5.0 \hat{k}$ and $C = 2.0 \hat{j}$. The value of $A \cdot (B \times C)$ is:? (Ans: -30)

$$B \times C = -5 \hat{k} \times 2 \hat{j} = -10 \hat{k} \times \hat{j} = -10 (-\hat{i}) = 10 \hat{i}$$

$$A \cdot (B \times C) = -3 \hat{i} \cdot 10 \hat{i} = \underline{\underline{-30}}$$

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- Q5. Two vectors A and B have magnitudes of 10 m and 15 m respectively. The angle between them is 65° . The component (projection) of B along A is:? (Ans: 6.3 m)



$$A = 10 \text{ m} \quad B = 15 \text{ m}$$

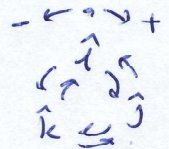
$$B \cdot \cos \theta = 15 \cdot \cos 65$$

$$= 15 \cdot 0.43 = 6.3 \text{ m}$$

- Q6. Let $A = 2.0i + 3.0k$ and $B = 2.0i + k$. The vector $D = (A - B) \times A$ is:? (Ans: ~~4~~ $0j$)

$$A - B = (2\hat{i} + 3\hat{k}) - (2\hat{i} + \hat{k}) = 2\hat{i} + 3\hat{k} - 2\hat{i} - \hat{k}$$

$$= 2\hat{k}$$



$$(A - B) \times A = 2\hat{k} \times (2\hat{i} + 3\hat{k}) = 4(+\hat{j}) + 0$$

$$= +4\hat{j}$$

- Q7. Three vectors A , B , and C are such that: $C = A + B$, $B = 5i$ and $C = 5j$. Find the angle between A and B : ...? (Ans: 135°)

$$\vec{C} = \vec{A} + \vec{B} \quad 5\hat{j} = \vec{A} + 5\hat{i} \quad \vec{A} = -5\hat{i} + 5\hat{j} \quad |\vec{A}| = \sqrt{5^2 + 5^2}$$

$$|\vec{A}| = \sqrt{50}$$

$$|\vec{B}| = 5$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$(-5\hat{i} + 5\hat{j}) \cdot (5\hat{i}) = \sqrt{50} \cdot 5 \cdot \cos \theta$$

$$-25 = \sqrt{50} \cdot 5 \cdot \cos \theta$$

$$\cos \theta = \frac{-5}{\sqrt{50}} = -0.71 \quad \theta = 135^\circ$$

- Q8. Three vectors are $A = 1.00i + 2.00j - 3.00k$, $B = 3.00k$ and $C = 6.00i - 7.00j$. Find $2C \cdot (A \times B)$. (Ans: 114)

$$A \times B = (\hat{i} + 2\hat{j} - 3\hat{k}) \times (3\hat{k}) = -3\hat{j} + 6\hat{i}$$

$$2C \cdot (A \times B) = 2[(6\hat{i} - 7\hat{j}) \cdot (-3\hat{j} + 6\hat{i})] = 2[36 + 21] = 2 \cdot 57 = \underline{114}$$

- Q9. Two vectors are given as: $A = -3.0i + 5.0j + 4.0k$ and $B = 4.0i + 5.0j + 3.0k$, where i, j and k are the unit vectors in the positive x, y and z directions. Find the angle between the vectors A and B . (Ans: 60 degrees)

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$|\vec{A}| = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50}$$

$$|\vec{B}| = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{50}$$

$$(-3\hat{i} + 5\hat{j} + 4\hat{k}) \cdot (4\hat{i} + 5\hat{j} + 3\hat{k}) = |\vec{A}| |\vec{B}| \cdot \cos \theta$$

$$(-12 + 25 + 12) = \sqrt{50} \sqrt{50} \cdot \cos \theta$$

$$25 = 50 \cos \theta \quad \cos \theta = \frac{1}{2}$$

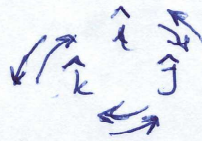
$$\theta = 60^\circ$$

KEY

Q10. In the cross product $F = v \times B$, take $v = 2.0 \hat{i}$, $F = 6.0 \hat{j}$ and the x component of vector B equals zero. What then is B in unit-vector notation? (Ans: $-3.0 \hat{k}$)

$$\frac{6 \hat{j}}{2} = \frac{2 \hat{i}}{2} \times B$$

$$3 \hat{j} = \hat{i} \times B$$



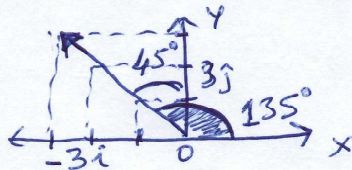
$$3 \hat{j} = \hat{i} \times B$$

$$\downarrow$$

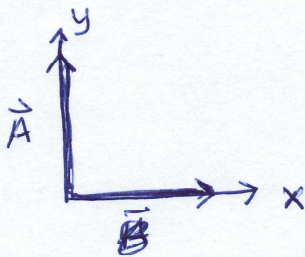
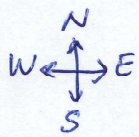
$$\underline{\underline{-3 \hat{k}}}$$

Q11. Two vectors are given by: $P = -1.5 \hat{i} + 2.0 \hat{j}$, $Q = 1.0 \hat{j}$. The angle that the vector $2P - Q$ makes with the positive x-axis is:....? (A: 135°)

$$2\vec{P} - \vec{Q} = 2(-1.5\hat{i} + 2\hat{j}) - (1\hat{j}) = -3\hat{i} + 4\hat{j} - 1\hat{j} = -3\hat{i} + 3\hat{j}$$



Q12. Vector \vec{A} is directed due north. Vector \vec{B} is directed due east. Determine the direction of the vector product, $\vec{A} \times \vec{B}$



$$\vec{A} = A_y \hat{j} \quad \vec{B} = B_x \hat{i}$$

$$\vec{A} \times \vec{B} = A_y \hat{j} \times B_x \hat{i} = A_y B_x (-\hat{k})$$

Q13. Under what conditions does $\vec{A} \times \vec{B} = AB$?

$$\vec{A} \times \vec{B} = A \cdot B \cdot \sin \theta$$

if $\vec{A} \times \vec{B} = A \cdot B$, then $\sin \theta = 1 \Rightarrow \theta = 90^\circ$.

This means that \vec{A} and \vec{B} are perpendicular to each other.

Q14. Let $A = 2.0\hat{i} - 3.0\hat{k}$ and $B = 2.0\hat{i} + \hat{k}$. Find the vector $D = (A - B) \times A = ?$

$$\vec{A} - \vec{B} = (2\hat{i} - 3\hat{k}) - (2\hat{i} + \hat{k}) = 2\hat{i} - 3\hat{k} - 2\hat{i} - \hat{k} = -4\hat{k}$$

$$D = (\vec{A} - \vec{B}) \times \vec{A} = -4\hat{k} \times (2\hat{i} - 3\hat{k}) = -8\hat{j} + 12 \cdot 0$$

$$\vec{D} = -8\hat{j}$$