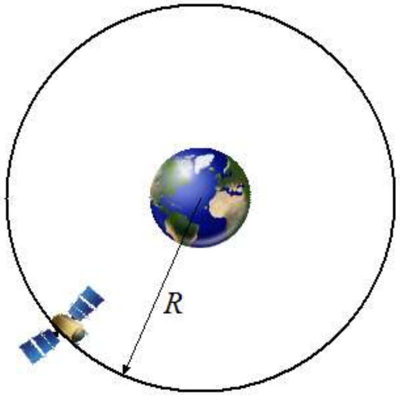
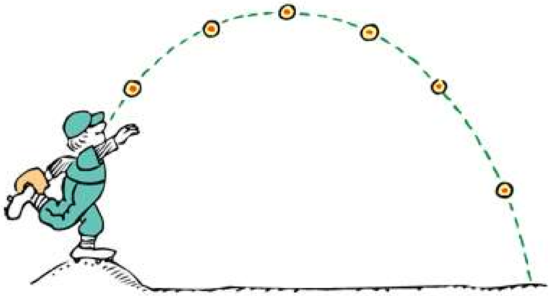
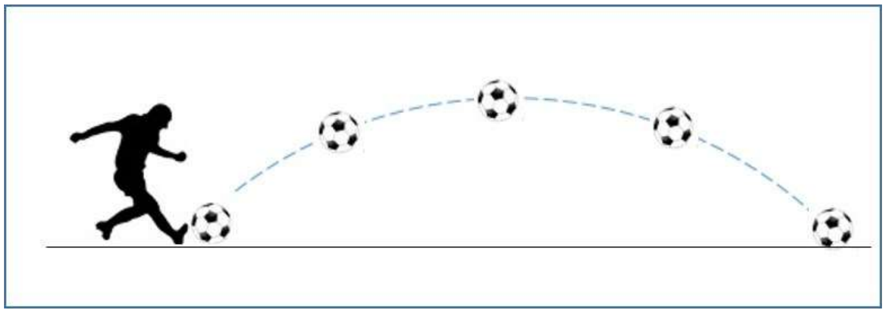


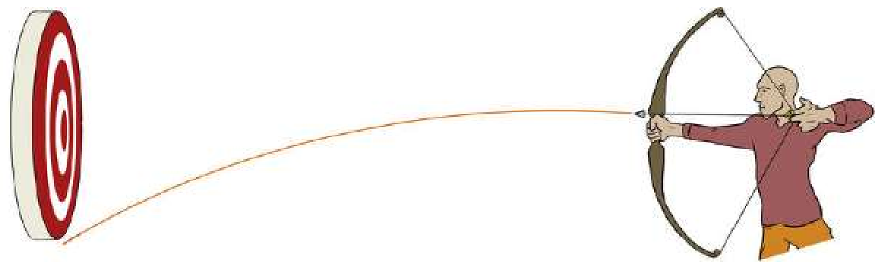
Chapter 4 – Part B
Motion in Two and Three Dimensions

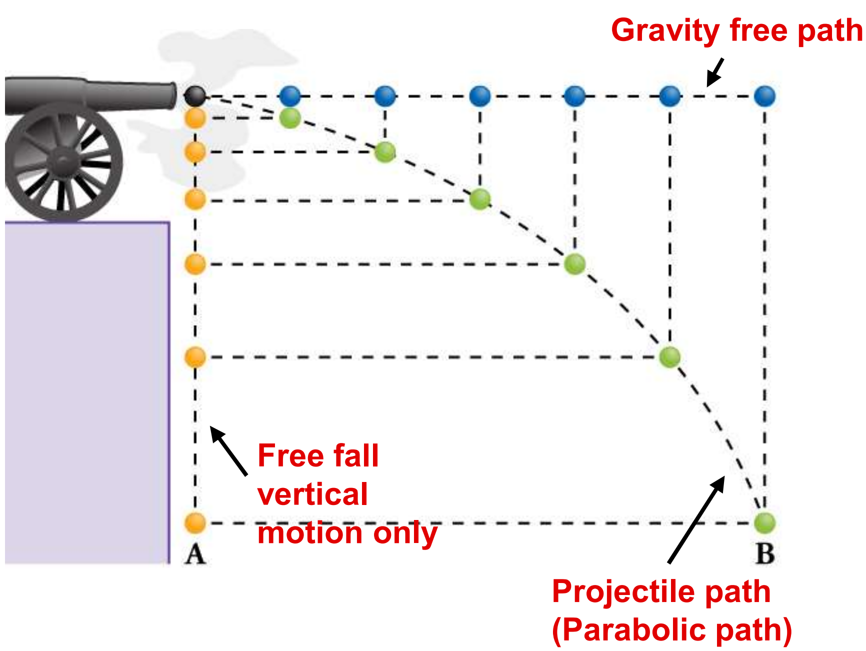
4-4 Projectile Motion
4-5 Uniform Circular Motion

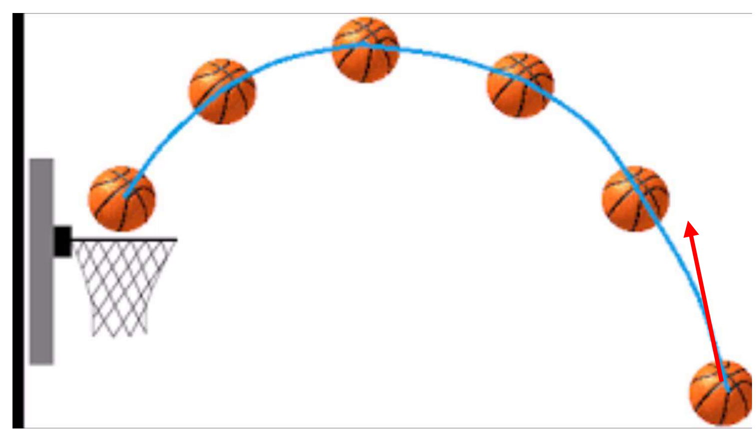


4-4 Projectile Motion
Projectile motion



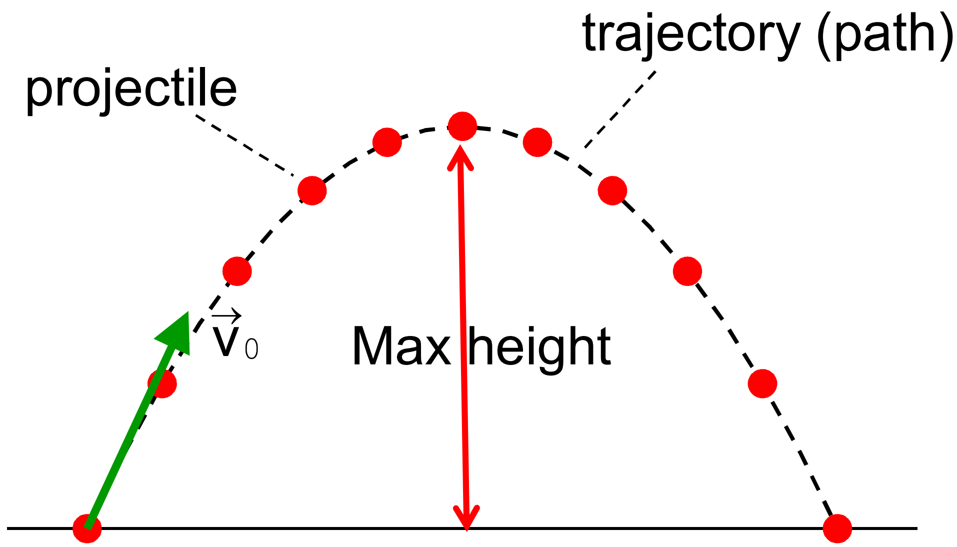






4-4 Projectile Motion

Projectile motion



A projectile is an object that is thrown or launched in air and moves only under the influence of Earth's gravity.

We will ignore air effects.

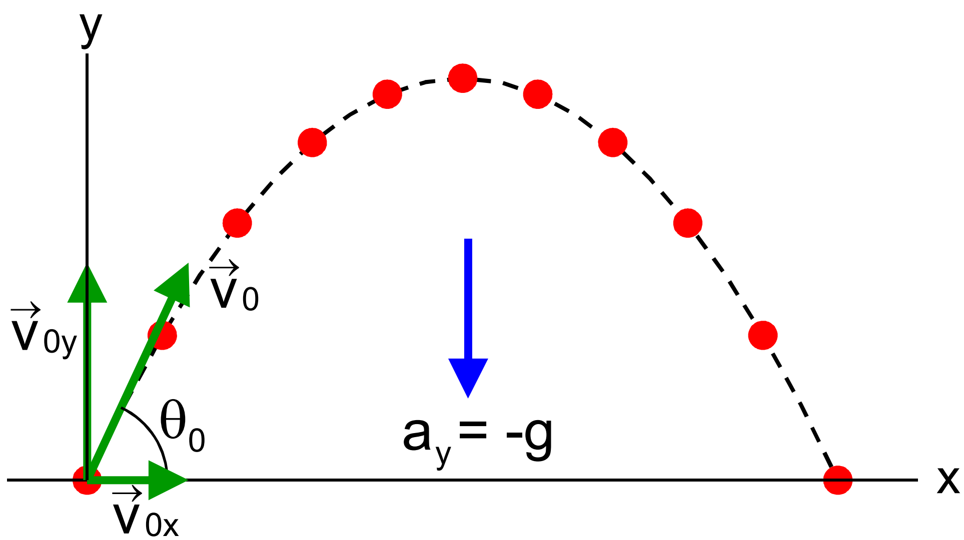
$a_x = 0$ The projectile has no acceleration in the horizontal direction.

$a_y = -g$

The projectile's acceleration is the free-fall acceleration

4-4 Projectile Motion

Equations of motion



$$\vec{v} = \vec{v}_0 + \vec{a}\, t$$

$$\vec{r} - \vec{r}_0 = \vec{v}_0\, t + \frac{1}{2}\vec{a}\, t^2$$

The horizontal motion

$a_x = 0$

$v_{0x} = v_0 \cos\theta_0$

$v_x = v_{0x}$

$x - x_0 = v_{0x}\, t$

The vertical motion

$a_y = -g$

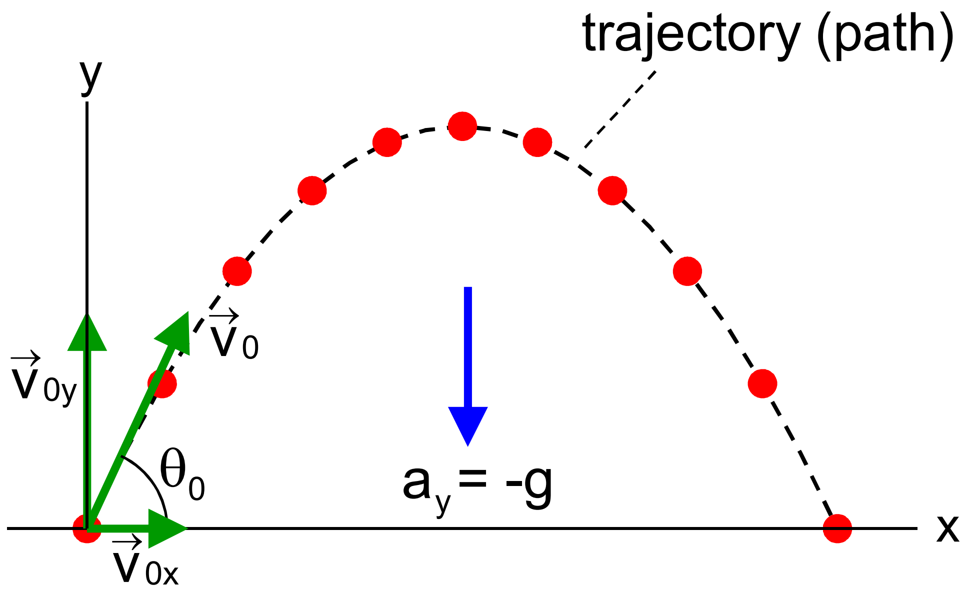
$v_{0y} = v_0 \sin\theta_0$

$v_y = v_{0y} - g\, t$

$y - y_0 = v_{0y}\, t - \frac{1}{2} g\, t^2$

4-4 Projectile Motion

The equation of the path



$$y = \tan\theta_0 \, x - \frac{g}{2(v_0\cos\theta_0)^2} x^2$$

The trajectory is a parabola

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 \rightarrow y = v_{0y} t - \frac{1}{2} g t^2 \rightarrow y = v_{0y} \frac{x}{v_{0x}} - \frac{1}{2} g \left(\frac{x}{v_{0x}}\right)^2$$

Set $x_0 = y_0 = 0$

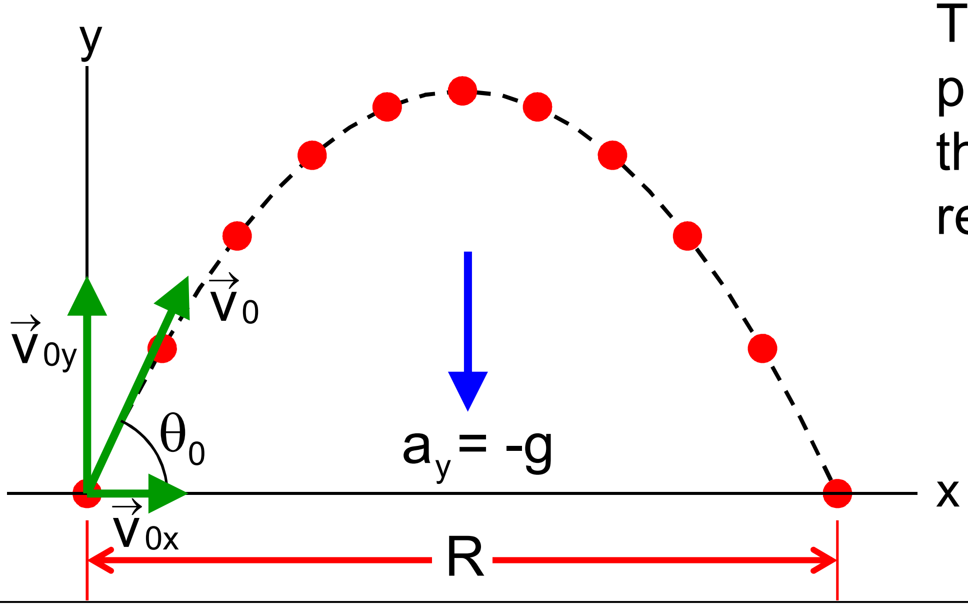
$$x - x_0 = v_{0x} t \rightarrow x = v_{0x} t \rightarrow t = \frac{x}{v_{0x}}$$

$$v_{0y} = v_0 \sin\theta_0$$

$$v_{0x} = v_0 \cos\theta_0$$

$$y = \tan\theta_0 \, x - \frac{g}{2(v_0\cos\theta_0)^2} x^2$$

4-4 Projectile Motion
The horizontal range



The horizontal range R of the projectile is the horizontal distance the projectile has traveled when it returns to its initial launch height.

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

$$y - y_0 = v_{0y}\, t - \frac{1}{2} g t^2 \xrightarrow{y - y_0 = 0} 0 = v_{0y}\, t - \frac{1}{2} g t^2 \rightarrow \frac{2v_{0y}}{g} = t$$

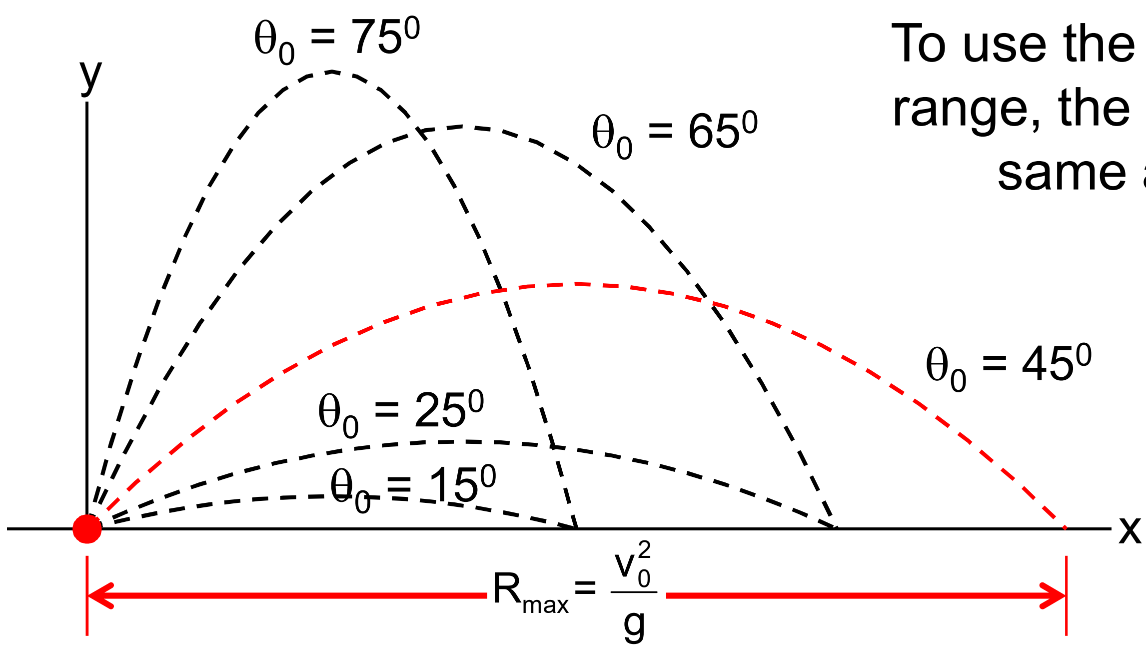
$$x - x_0 = v_{0x}\, t \xrightarrow{x - x_0 = R} R = v_{0x}\, t \rightarrow R = v_{0x} \frac{2v_{0y}}{g}$$

$$v_{0y} = v_0 \sin\theta_0$$

$$v_{0x} = v_0 \cos\theta_0$$

$$R = \frac{2v_0^2}{g} \sin\theta_0 \cos\theta_0 \rightarrow R = \frac{v_0^2}{g} \sin 2\theta_0$$

4-4 Projectile Motion
The horizontal range



To use the formula for the horizontal range, the final height should be the same as the launch height.

$a_y = -g$

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

The horizontal range is maximum for a launch angle of 45^0

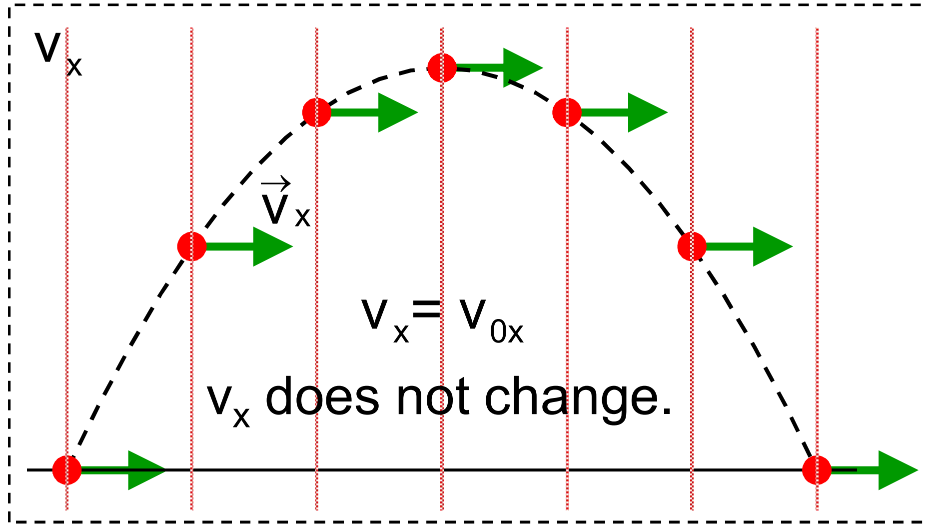
The horizontal range is maximum when $\sin 2\theta_0 = 1$

$\rightarrow 2\theta_0 = 90^0$ or $\theta_0 = 45^0$.

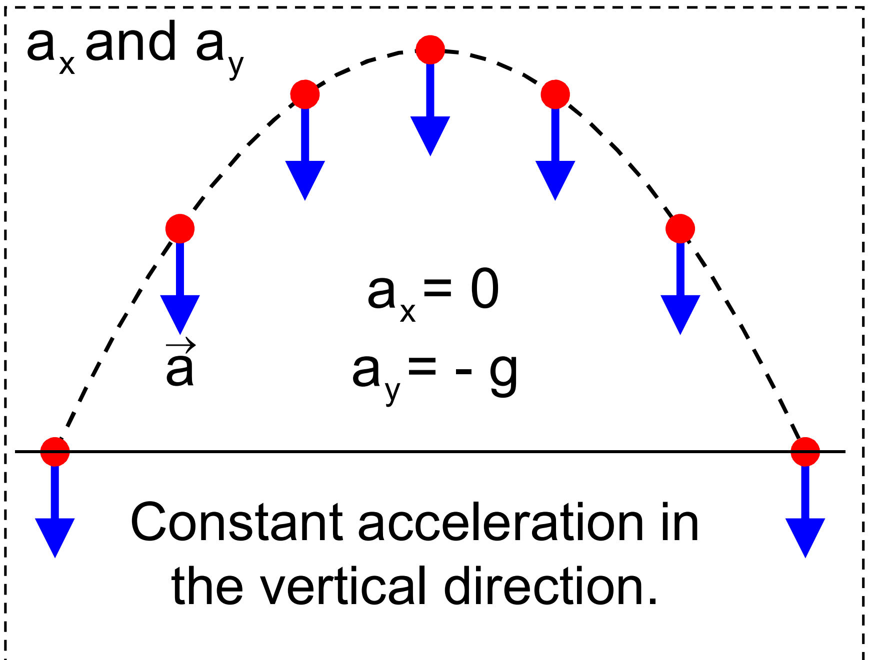
4-4 Projectile Motion

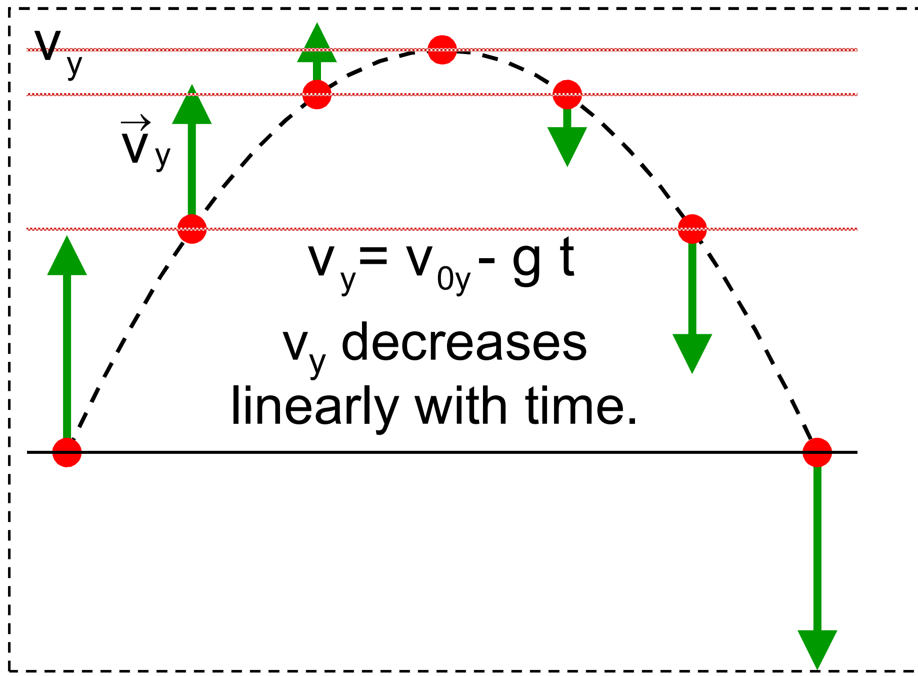
Checkpoint

During the flight of a projectile, what happens to its
a_x,
a_y,
v_x, and
v_y?



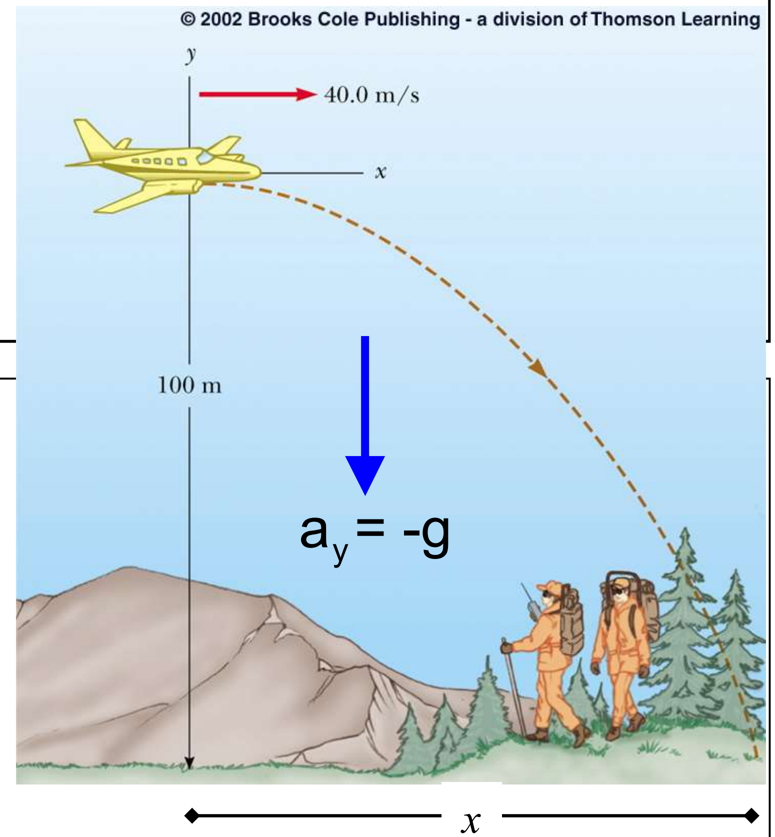
Solution





4-4 Projectile Motion
Example

A plane drops a package of emergency to explorers at the top of a hill. The plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground. Where does the package strike the ground relative to the point at which it was released?



Solution

velocity: v=40.0 m/s

height: h=100 m

Distance d=?

1. Introduce coordinate frame:
   Oy: y is directed up
   Ox: x is directed right

2. Note: $v_{ox} = v = +40$ m/s, $v_{oy} = 0$ m/s

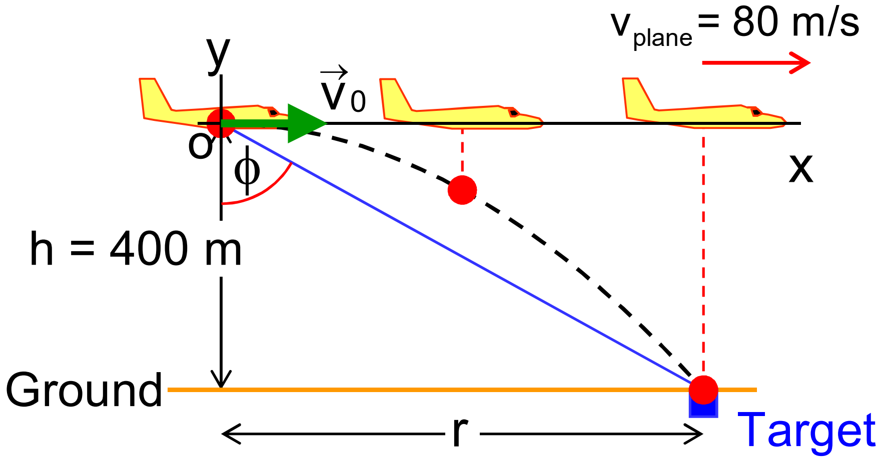
$$y - y_0 = v_{0y}\, t - \frac{1}{2} g t^2 \quad (y = 0,\ v_{0y} = 0) \longrightarrow -h = -\frac{1}{2} g t^2 \longrightarrow t = \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{2(-100\,m)}{-9.8\,m/s^2}} = 4.51\,s$$

$$\underline{Ox}: x = v_{x0} t,\ so \quad x = (40\,m/s)(4.51\,s) = \underline{180\,m}$$

4-4 Projectile Motion

Example



A plane flies at 80 m/s and constant elevation of 400 m. **The plane releases an object, the object hits the target.** To hit a target on the ground, what should be the angle ϕ of the pilot's line of sight to the target when the release is made?

Solution

Choose the origin at the release point → $x_0 = y_0 = 0$

Since the object is released, its initial velocity = airplane's velocity;

$v_{0x} = v_0 = 80$ m/s

$v_{0y} = 0$

$$\phi = \tan^{-1}\frac{r}{h}$$

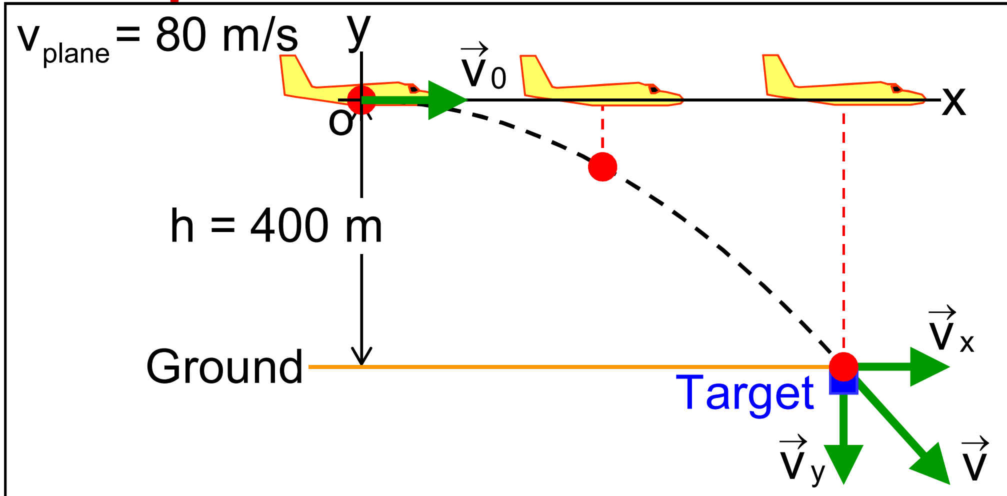
$$x - x_0 = v_{0x}\,t \rightarrow r = v_0\,t$$

$$y - y_0 = v_{0y}\,t - \frac{1}{2}g\,t^2 \rightarrow -h = -\frac{1}{2}g\,t^2 \rightarrow t = \sqrt{\frac{2h}{g}} \rightarrow r = v_0\sqrt{\frac{2h}{g}}$$

$$\phi = \tan^{-1}\frac{v_0\sqrt{2h/g}}{h} = \tan^{-1}\frac{v_0\sqrt{2}}{\sqrt{gh}} = \tan^{-1}\frac{(80\text{ m/s})\sqrt{2}}{\sqrt{(9.8\text{ m/s}^2)(400\text{ m})}} = 61^\circ$$

4-4 Projectile Motion

Example



As the object reaches the target, calculate the velocity in unit-vector notation and magnitude angle notation?

Solution

From previous example

$$t = \sqrt{\frac{2h}{g}} \qquad v_{0x} = v_0 = 80 \text{ m/s} \qquad v_{0y} = 0$$

$$v_x = v_{0x} = 80 \text{ m/s}$$

$$v_y = v_{0y} - g\,t = -g\,t = -g\sqrt{\frac{2h}{g}} = -\sqrt{2hg} = -\sqrt{2\,(400\text{ m})(9.8\text{ m/s}^2)}$$

$$= -89 \text{ m/s}$$

Velocity in unit-vector notation $\quad \vec{v} = (80\text{ m/s})\hat{i} - (89\text{ m/s})\hat{j}$

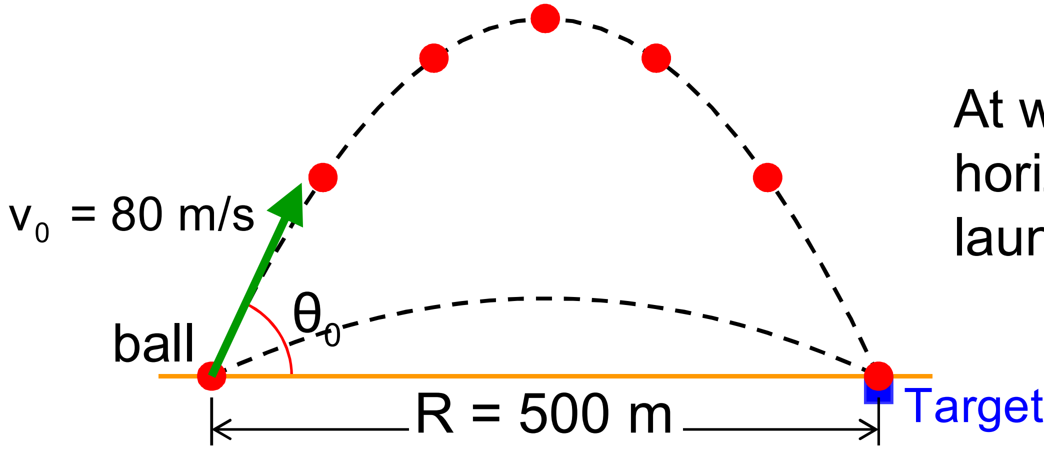
Velocity in magnitude-angle notation

$$v = \sqrt{(80\text{ m/s})^2 + (-89\text{ m/s})^2} = 120 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{-89\text{ m/s}}{80\text{ m/s}} = -48°$$

4-4 Projectile Motion

Example



At what angle from the horizontal must a ball be launched to hit the target?

Solution

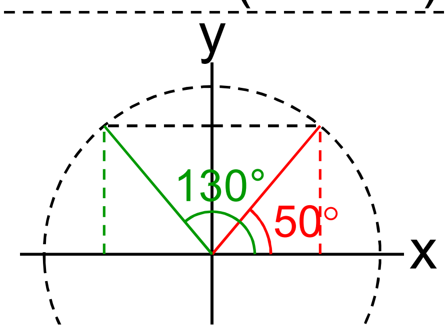
Since the final height is the same as the launch height, we can use

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

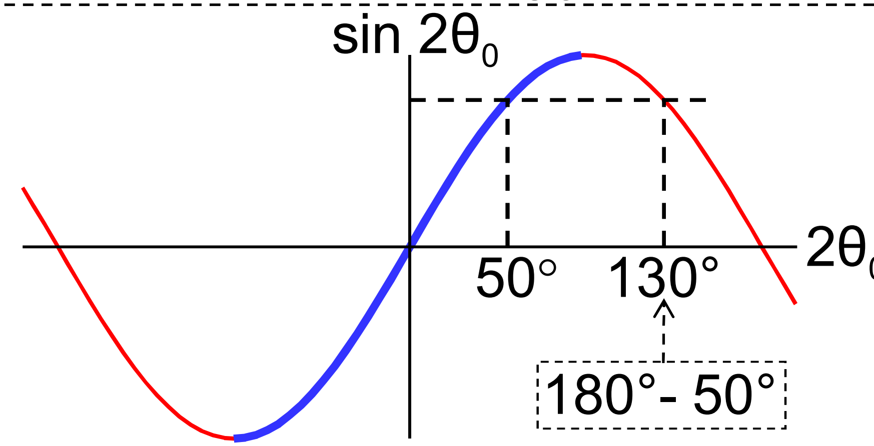
$$2\theta_0 = \sin^{-1}\frac{R\,g}{v_0^2} = \sin^{-1}\frac{(500\text{ m})(9.8\text{ m/s}^2)}{(80\text{ m/s})^2} = \sin^{-1} 0.766 = \begin{cases} 50^\circ \\ 130^\circ \end{cases}$$

Using a calculator (50°)

$$\theta_0 = \begin{cases} 25^\circ \\ 65^\circ \end{cases}$$

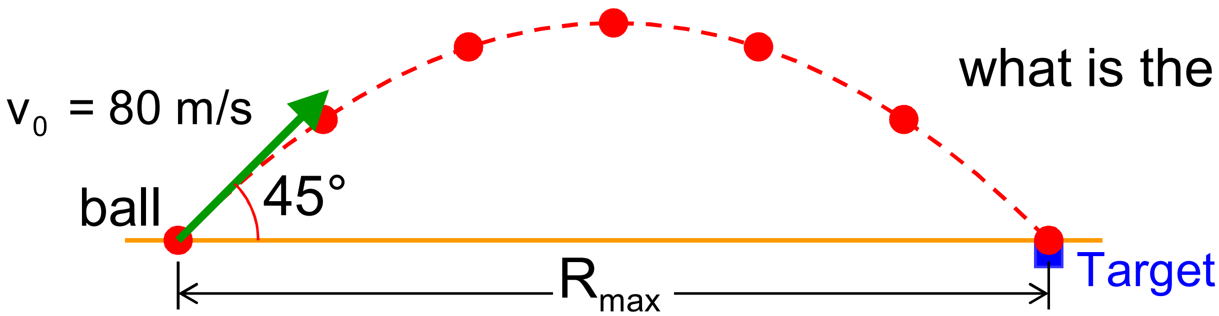


If radius = 1,
x = cos θ and y = sin θ



4-4 Projectile Motion

Example



what is the maximum range?

Solution The maximum range corresponds to elevation angle θ_0 of 45^0

$$R_{max} = \frac{v_0^2}{g} \sin 2(45^\circ) = \frac{v_0^2}{g} = 653 \text{ m} \approx 650 \text{ m}$$

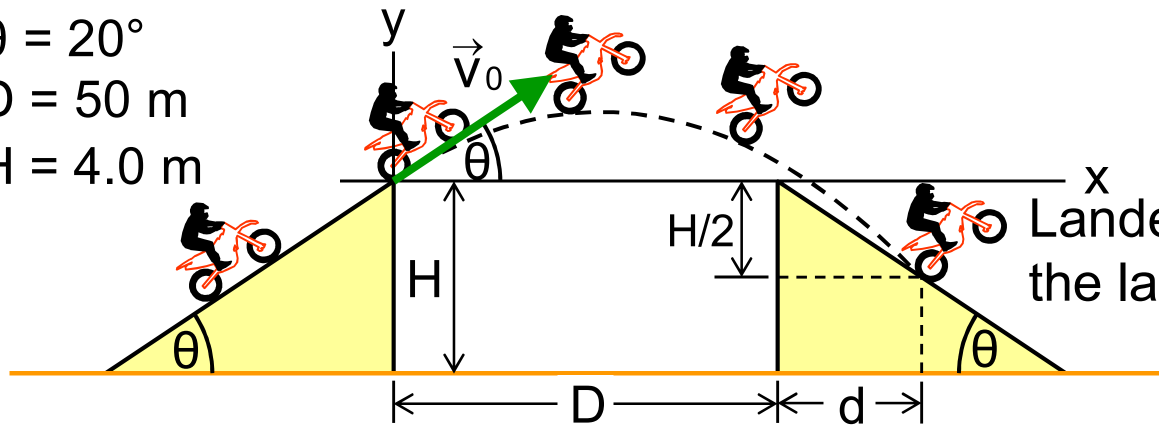
4-4 Projectile Motion

Example

$\theta = 20°$

$D = 50\ \text{m}$

$H = 4.0\ \text{m}$



Landed ½ way down the landing ramp.

What is the speed he left the launch ramp?

Solution Choose the origin at the launch point → $x_0 = y_0 = 0$

$$x = v_{0x}\, t \quad\rightarrow\quad t = \frac{x}{v_{0x}}$$

$$y = v_{0y}\, t - \frac{1}{2} g t^2 \quad\rightarrow\quad y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{1}{2} g \left(\frac{x}{v_{0x}}\right)^2$$

$$v_{0y} = v_0 \sin\theta_0$$

$$v_{0x} = v_0 \cos\theta_0$$

$$\theta_0 = \theta$$

$$y = x\tan\theta - \frac{1}{2} g \left(\frac{x}{v_0\cos\theta}\right)^2$$

Solve for v_0

$$y = -\frac{H}{2} = -2.0\ \text{m}$$

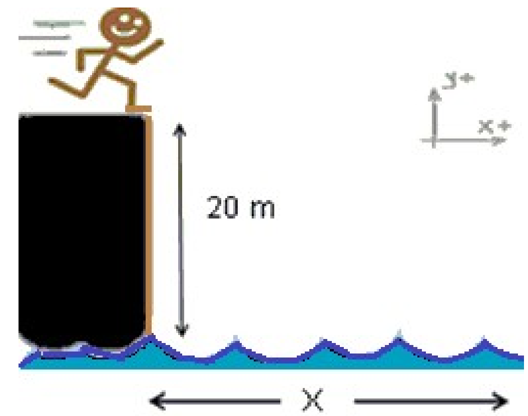
$$d = \frac{H/2}{\tan\theta} = 5.0\ \text{m}$$

$$x = D + d = 55\ \text{m}$$

$$v_0 = \frac{x}{\cos\theta}\sqrt{\frac{g}{2\,(x\tan\theta - y)}} = \frac{55\ \text{m}}{\cos 20°}\sqrt{\frac{9.8\ \text{m/s}^2}{2\,((55\ \text{m})\tan 20° - (-2.0\ \text{m}))}} = 28\ \text{m/s}$$

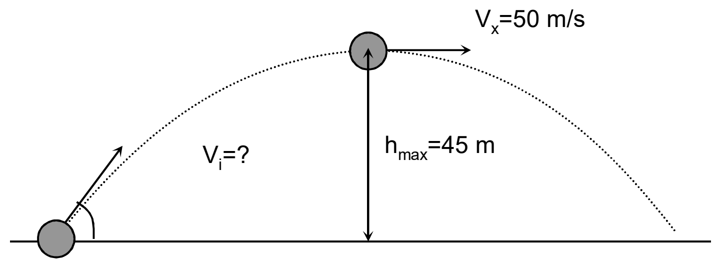
4-4 Projectile Motion
Questions

1. A boy is running on cliff to jump into the sea. The height of the cliff is 20 m. The speed of the boy is 10 m/s. (Take g = - 10 m/s^2)
   a) Find the time for the boy to touch into the sea.
   b) Find the vertical velocity of the boy when he touches on the sea.
   c) Find the horizontal distance (X) of the boy when he touches on the sea.

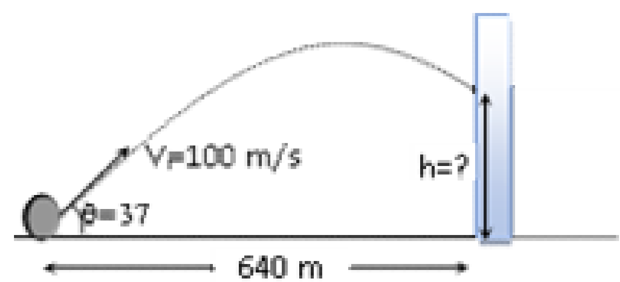


2. A ball is thrown at an angle 30^o with the horizontal. If the initial velocity is 20 m/s,
   a) Find its max height (How high will it go?)
   b) The distance that it hits the ground (Range)?
   c) How long will it take for the arrow to strike the ground? (Take g = 10 m/s^2)

3. According to the figure;
   a) What is the initial velocity of the projectile?
   b) Find the time of flight.



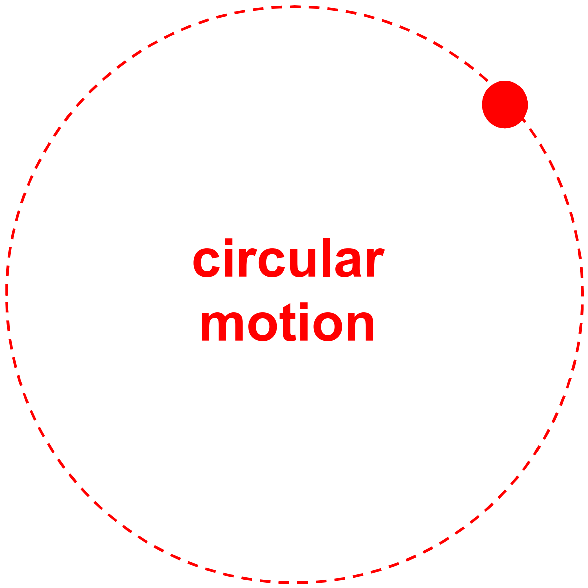
4. Look at the figure. At what height does the projectile hit the wall?



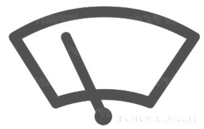
4-5 Uniform Circular Motion

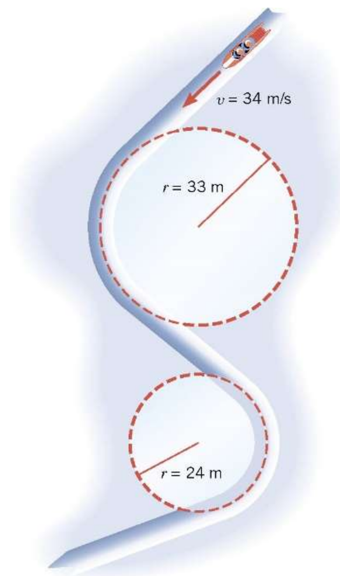
Circular motion

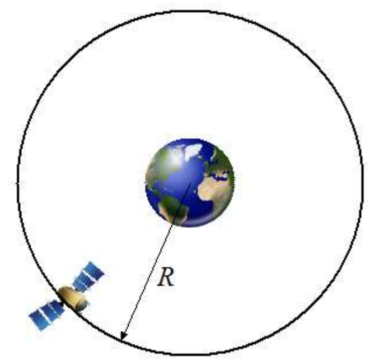
An object moving around a circular path makes **circular motion**.

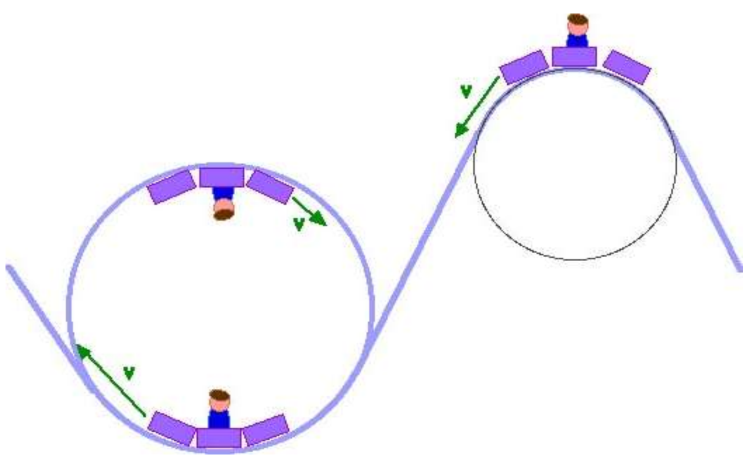


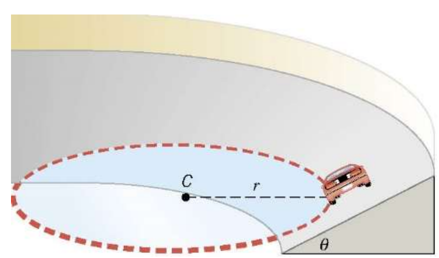
windshield wiper



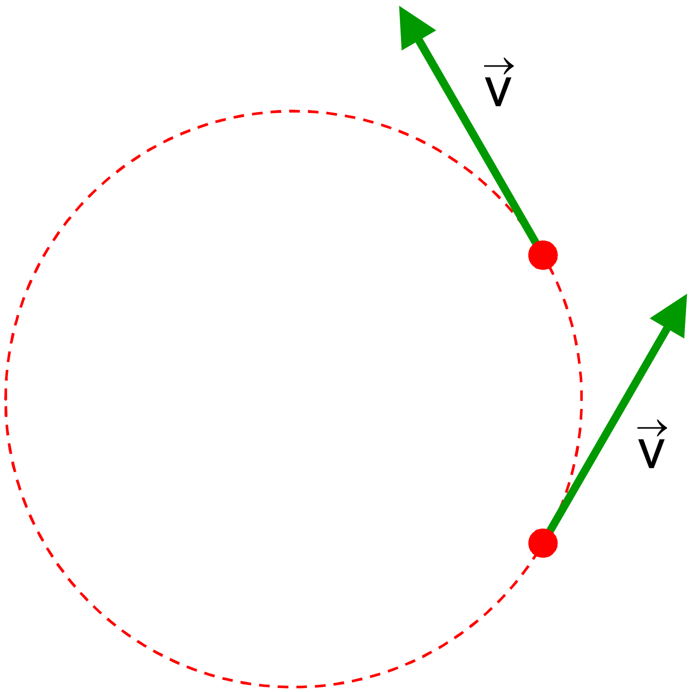








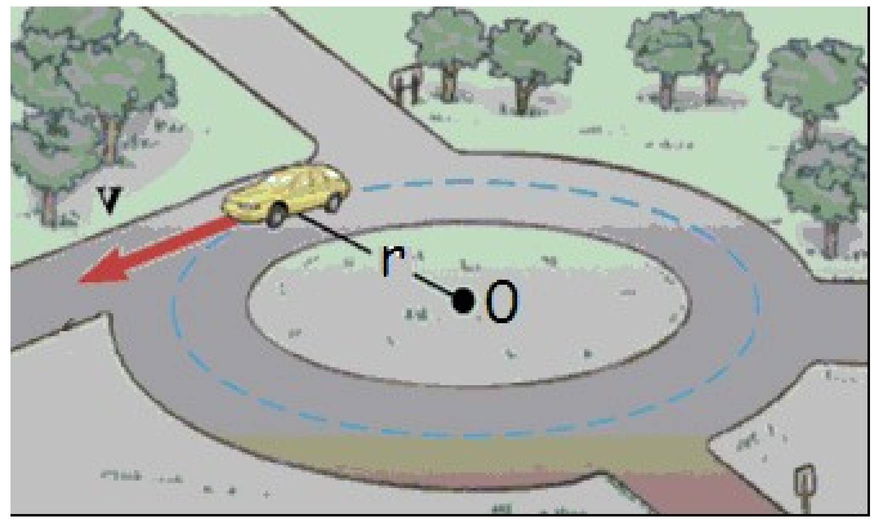
4-5 Uniform Circular Motion
Acceleration and velocity direction



Uniform motion = constant speed.

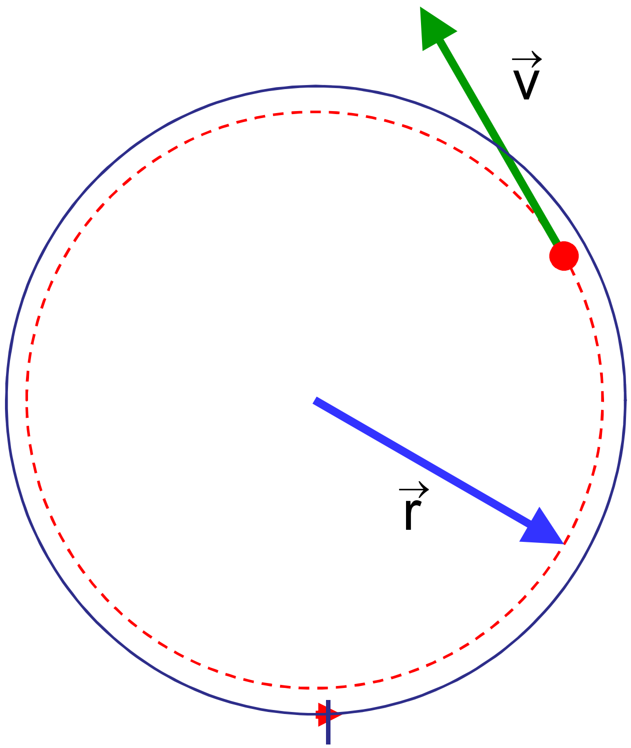
A particle moves in a circular path at a **constant** speed.

The magnitude of its velocity (speed) does not change.
The direction of its velocity changes.



Although its speed is constant, the particle accelerates because of the change in the direction of its velocity.

4-5 Uniform Circular Motion
Acceleration and velocity direction



T : Period

The period is the time the particle takes to make one revolution.

The distance traveled by the particle in one revolution
= the circumference of the circle
= $2\pi r$

Period

$$T = \frac{2\pi r}{v}$$

$2\pi r$ — radius of the circle

v — speed of the particle

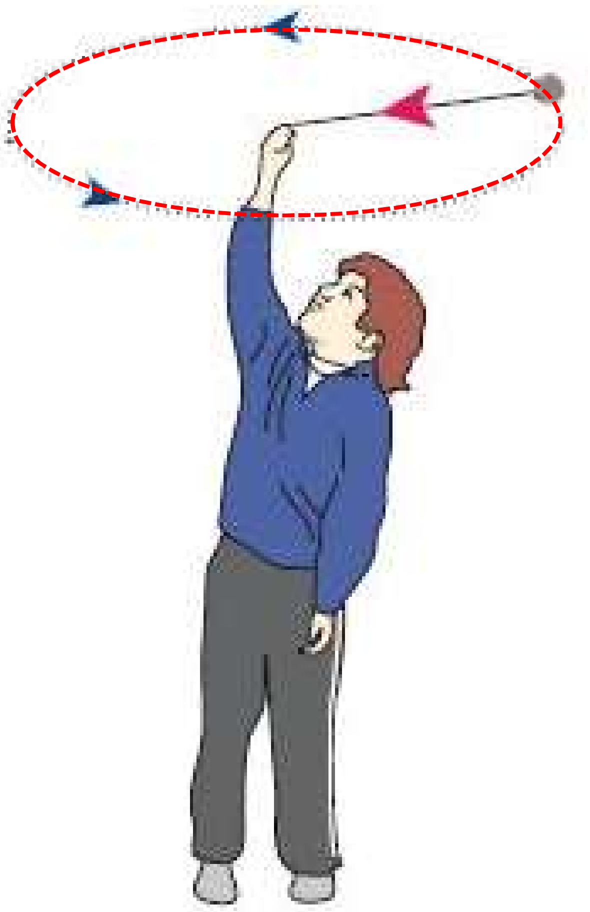
SI units

$$T = \frac{2\pi r}{v}$$

$$\frac{m}{\left(\frac{m}{s}\right)} = s$$

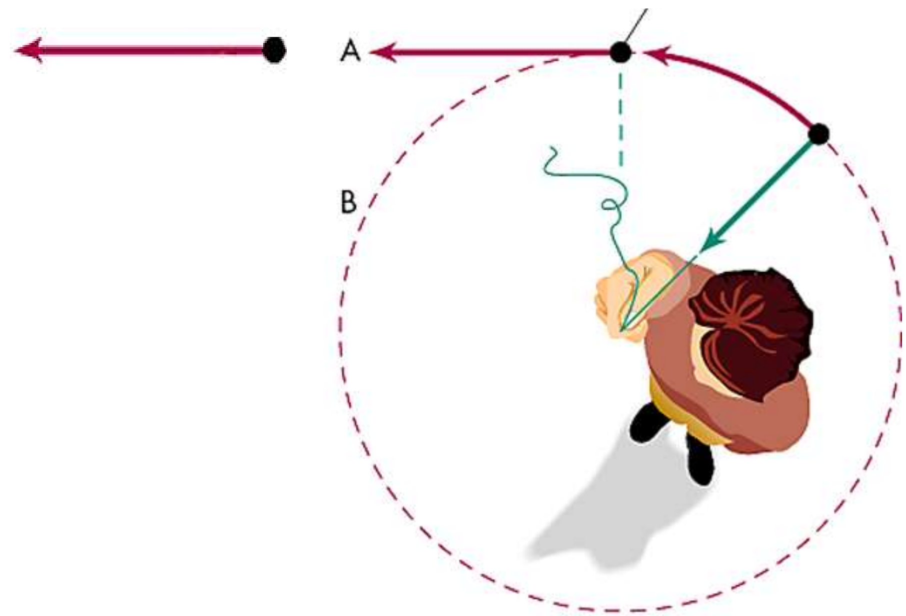
4-5 Uniform Circular Motion
Acceleration and velocity direction

Is the ball accelerating ?



The object then moves along a straight line.

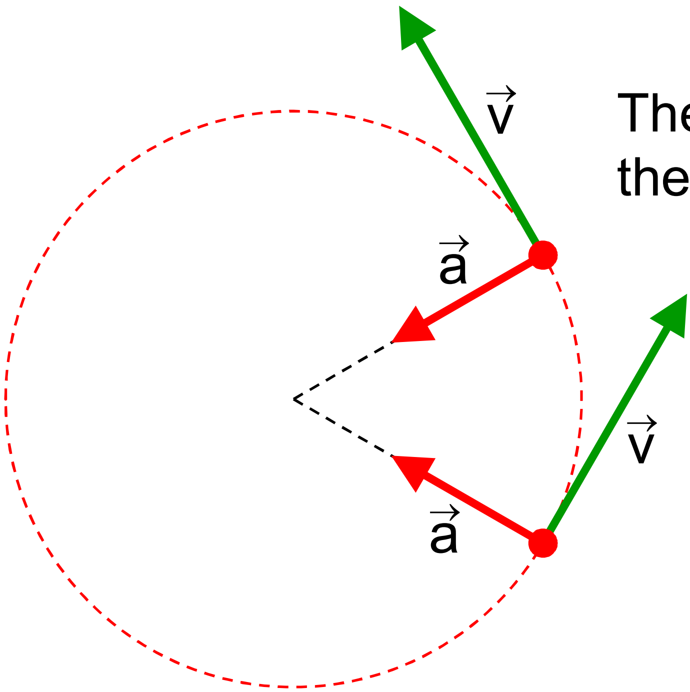
The string breaks at this position



Without tension the object moves in a straight line.

Tension changes the direction of the object.

4-5 Uniform Circular Motion
Centripetal acceleration



The velocity is always directed tangent to the circle in the direction of motion.

Centripetal acceleration

$$a = \frac{v^2}{r}$$

v — speed of the particle

r — radius of the circle

SI units

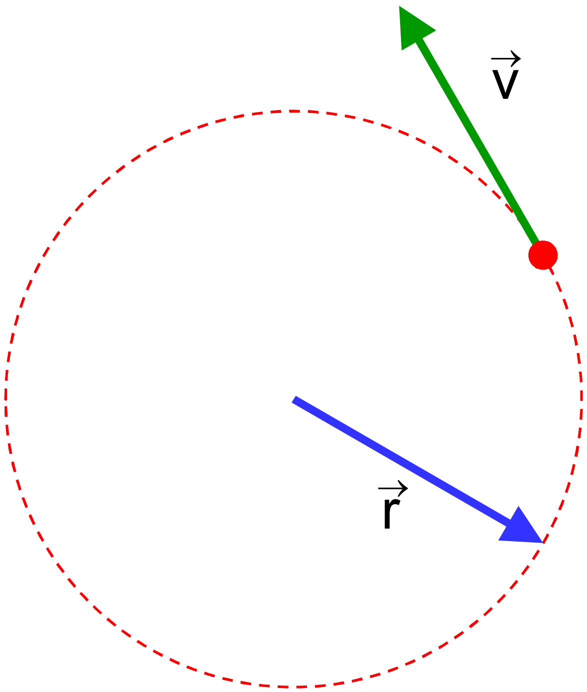
$$a = \frac{v^2}{r} \qquad \frac{\left(\frac{m}{s}\right)^2}{m} = \frac{m}{s^2}$$

The acceleration is always directed radially inward.

It is called a centripetal acceleration.

Centripetal means **center seeking.**

4-5 Uniform Circular Motion
Period of revolution



The period is the time the particle takes to make one revolution.

The distance traveled by the particle in one revolution
= the circumference of the circle
= $2\pi r$

Period

$$T = \frac{2\pi r}{v}$$

radius of the circle

speed of the particle

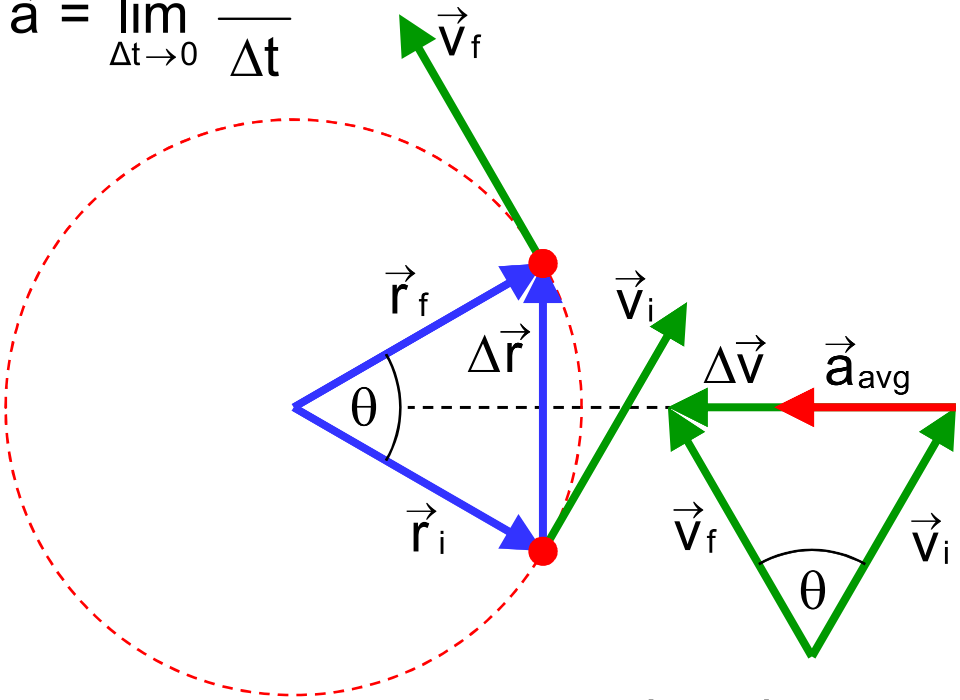
SI units

$$T = \frac{2\pi r}{v}$$

$$\frac{m}{\left(\frac{m}{s}\right)} = s$$

4-5 Uniform Circular Motion
Derivation - Centripetal acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$



a_{avg} has the same direction as Δv, and points toward the center.

The position-vector and velocity-vector triangles are similar, because the angle between their equal sides are the same.

$$\frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r}$$

$$|\vec{v}_i| = |\vec{v}_f| = v$$

$$|\vec{r}_i| = |\vec{r}_f| = r$$

$$|\Delta \vec{v}| = v \frac{|\Delta \vec{r}|}{r}$$

$$\frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{r} \frac{|\Delta \vec{r}|}{\Delta t}$$

$$|\vec{v}| = v$$

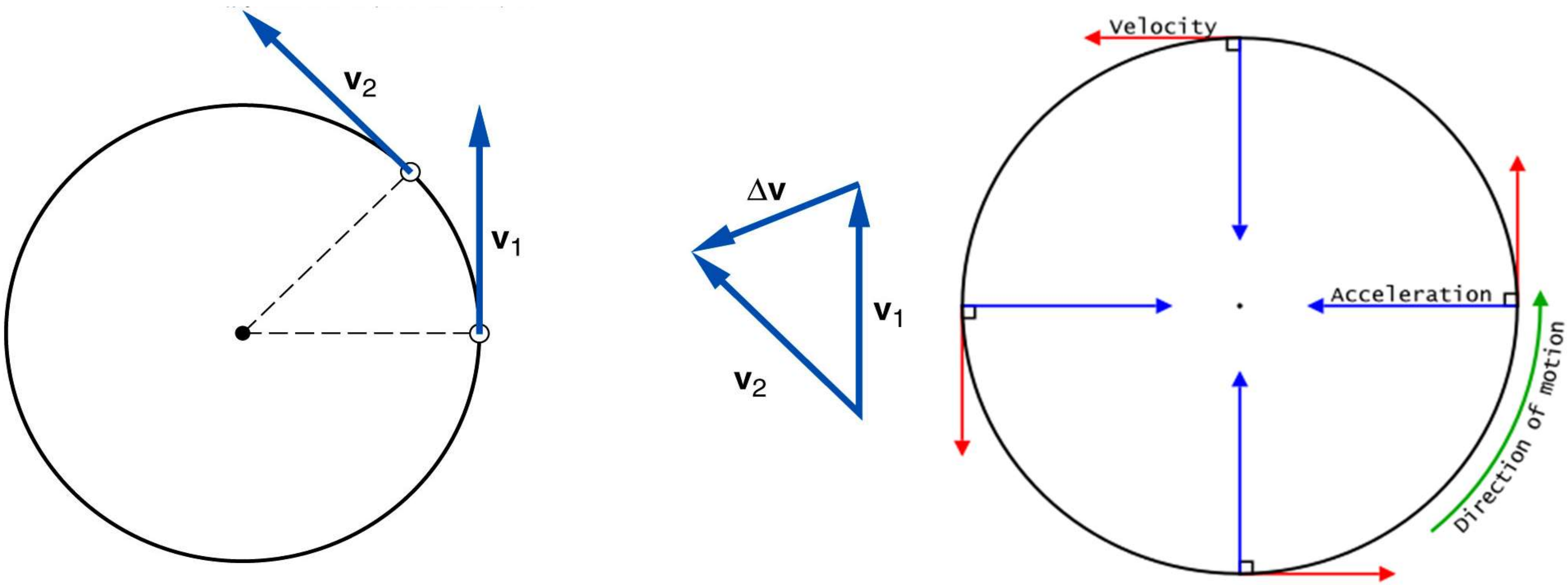
$$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{r} \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t}$$

$$= \frac{v}{r} v$$

$$a = \frac{v^2}{r}$$

4-5 Uniform Circular Motion

Circular Motion → Accelerating Motion



- Object moves along a circular path.
- The direction of velocity is always changing.
- Changing velocity implies that the object accelerates.

4-5 Uniform Circular Motion

Example

What is the centripetal acceleration, in g units, of a particle moving at speed of v = 1000 km/h in a circular path of radius r = 10.0 km?

Solution

$$a = \frac{v^2}{r} = \frac{(1000\ \text{km/h})^2}{(10.0\ \text{km})} = 1.00 \times 10^5\ \frac{\text{km}}{\text{h}^2}$$

$$= (1.00 \times 10^5\ \frac{\text{km}}{\text{h}^2})(\frac{1000\ \text{m}}{\text{km}})(\frac{1\text{h}}{3600\ \text{s}})^2 = 7.72\ \frac{\text{m}}{\text{s}^2}$$

$$= (7.72\ \text{m/s}^2)(\frac{\text{g}}{9.80\ \text{m/s}^2}) = 0.787\ \text{g}$$

4-5 Uniform Circular Motion
Checkpoint

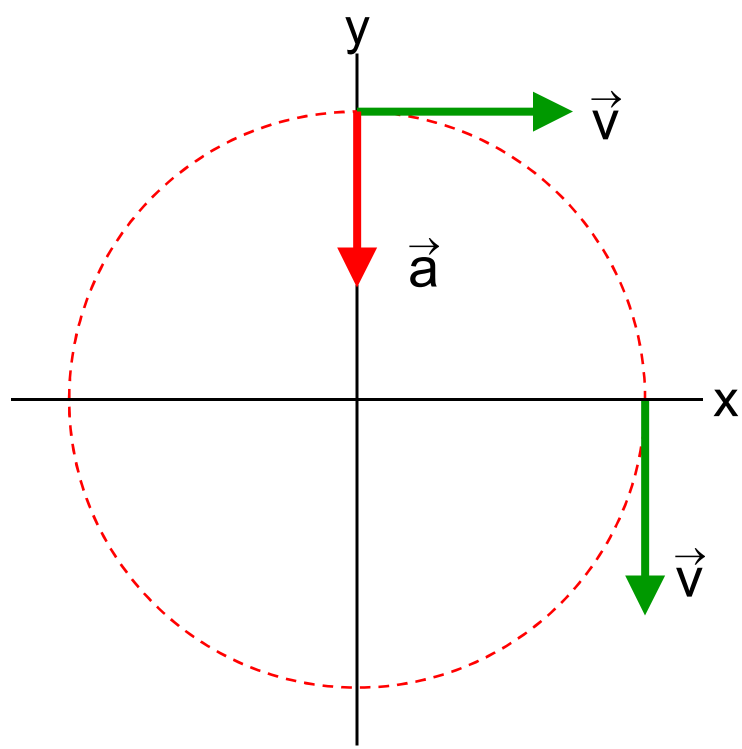
A particle moves at a constant speed along a circular path in an xy plane, with the center located at the origin.
When the particle at x = 10 m, its velocity is $-(5.0\ \text{m/s})\hat{j}$.
What is the particle's velocity and acceleration when it is at y = 10 m.

Solution

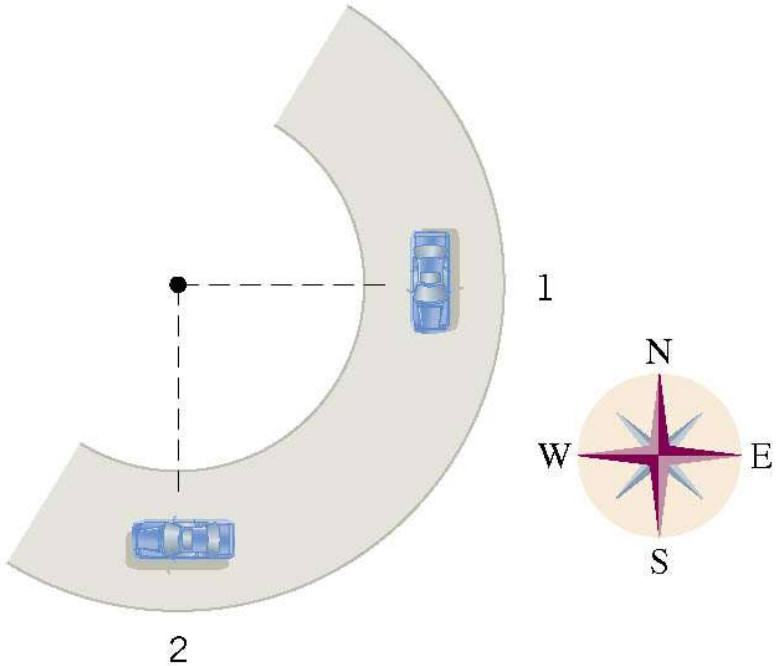
$$\vec{v} = (5\ \text{m/s})\hat{i}$$

$$a = \frac{v^2}{r} = \frac{(5\ \text{m/s})^2}{(10\ \text{m})} = 2.5\ \text{m/s}^2$$

$$\vec{a} = -(2.5\ \text{m/s}^2)\hat{j}$$



4-5 Uniform Circular Motion
Example



The car in the drawing is moving clockwise around a circular section of road at a constant speed.

What are the directions of its velocity and acceleration at

A) position 1 and

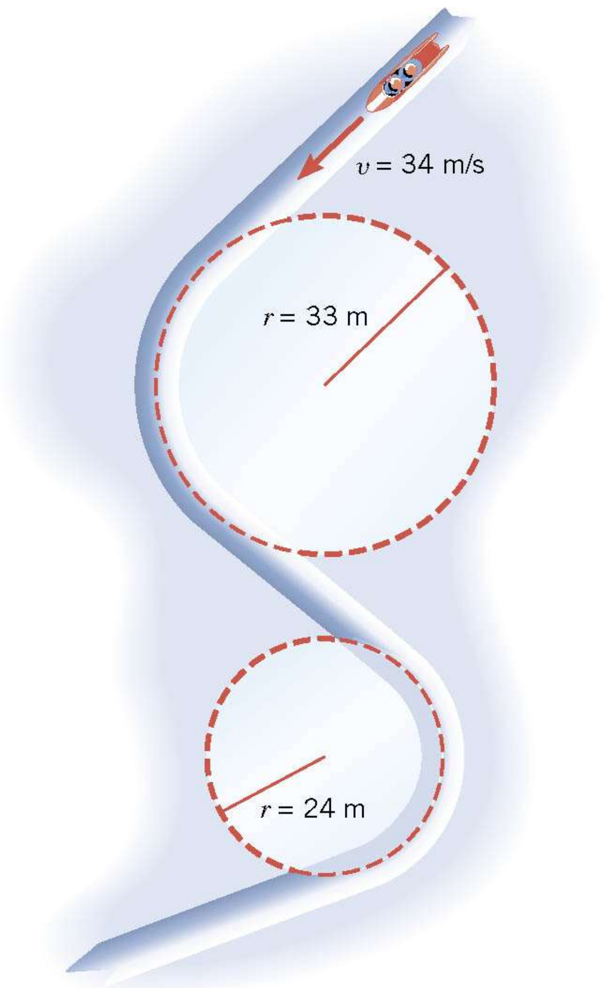
B) position 2?

Solution

A) The velocity is due south, and the acceleration is due west.

B) The velocity is due west, and the acceleration is due north.

4-5 Uniform Circular Motion
Example



A car turns with radii of 33 m and 24 m, as the figure illustrates.

Find the centripetal acceleration at each turn for a speed of 34 m/s.

Solution

From $a_c = v^2 / r$ it follows that

Radius = 33 m

$$a_c = \frac{(34m/s)^2}{33m} = 35m/s^2$$

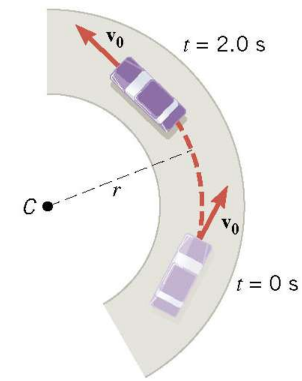
Radius = 24 m

$$a_c = \frac{(34m/s)^2}{24m} = 48m/s^2$$

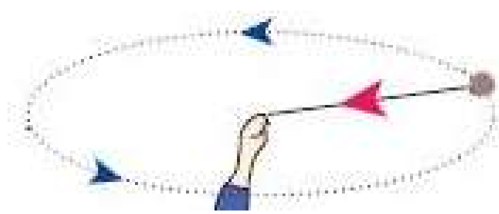
4-5 Uniform Circular Motion
Questions

1. The blade of a windshield wiper moves through an angle of 90 degrees in 0.28 seconds. The tip of the blade moves on the arc of a circle that has a radius of 0.76m.
What is the magnitude of the centripetal acceleration of the tip of the blade?

2. An automobile is traveling at a speed of 18 m/s in uniform circular motion as it makes a turn. It has a centripetal acceleration whose magnitude is 6 m/s^2. Calculate the radius if the path.



3. A ball rotates at a constant speed of 3 m/s on the end of 1.2 m long string. The string describes a horizontal circle.
Calculate the centripetal acceleration of the ball.



4. Look at the figure. The ball is making circular motion on x-y plane. Show the direction of centripetal acceleration and velocity of the object at points A, B, C and D.