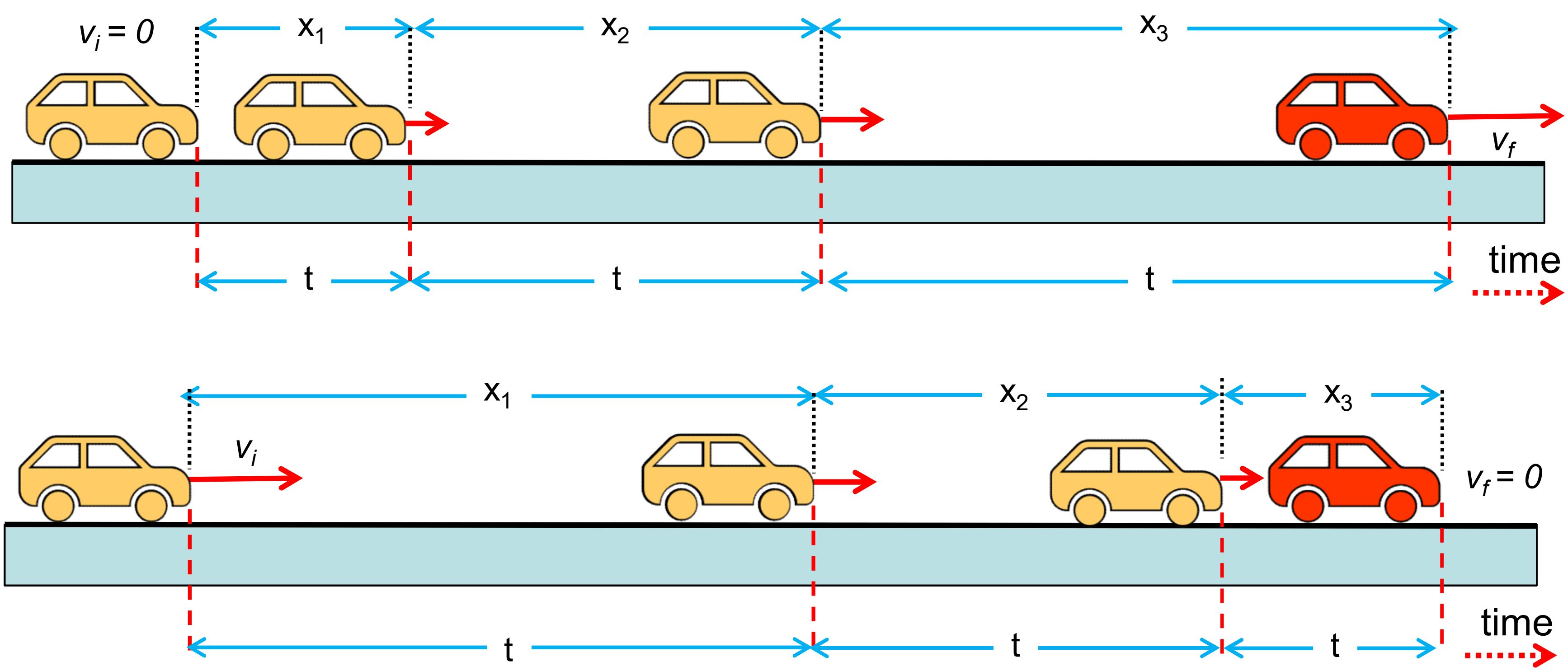
Chapter 3 - Part B
Motion Along a Straight Line

3-4 Acceleration
3-5 Constant Acceleration

3-4 Acceleration
Acceleration

When a car starts from rest and travels in a straight line at increasing speeds, it is *accelerating* in the direction of travel.



Acceleration is the rate of change of velocity of an object over a period of time. It means that velocity of an accelerating object is NOT-constant.

3-4 Acceleration
Acceleration

Changing velocity (non-uniform) means an acceleration is present.

The average acceleration a_{avg} over a time interval Δt is:

$$\vec{a}_{average} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

v_f

Average acceleration is a vector quantity (described by both magnitude and direction).

When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing.

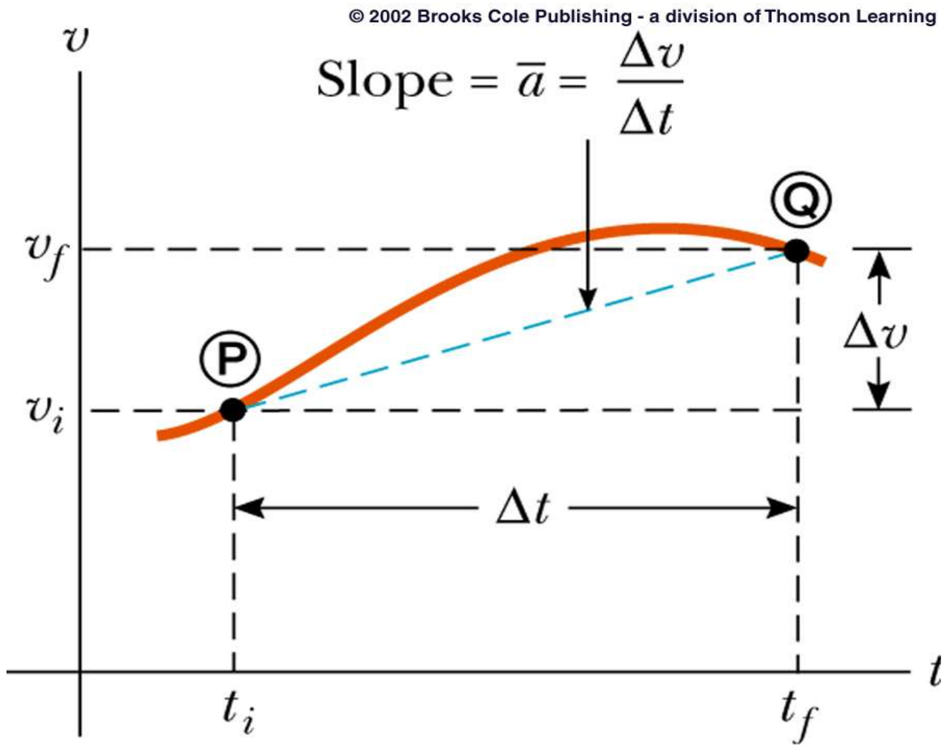
When the sign of the velocity and the acceleration are opposite, the speed is decreasing.

The SI unit for acceleration is meter per second squared-m/s^2.

3-4 Acceleration
Acceleration is the slope of v-t curve

Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph.

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$



Uniform or *constant acceleration* is a type of motion in which the velocity of an object changes by an equal amount in every equal time period.

3-4 Acceleration
Definitions

The instantaneous acceleration a is:

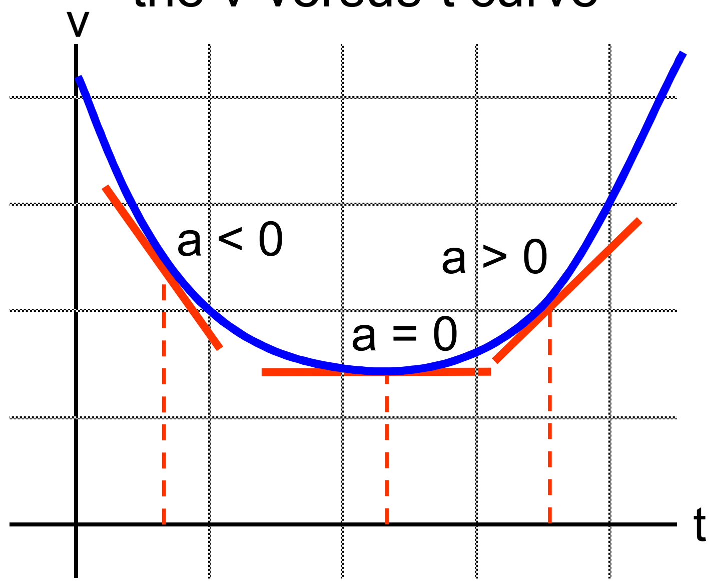
$$a = \frac{dv}{dt} = \text{the slope of the v-versus-t curve.}$$

The acceleration of a particle is the second derivative of its position with respect to time.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

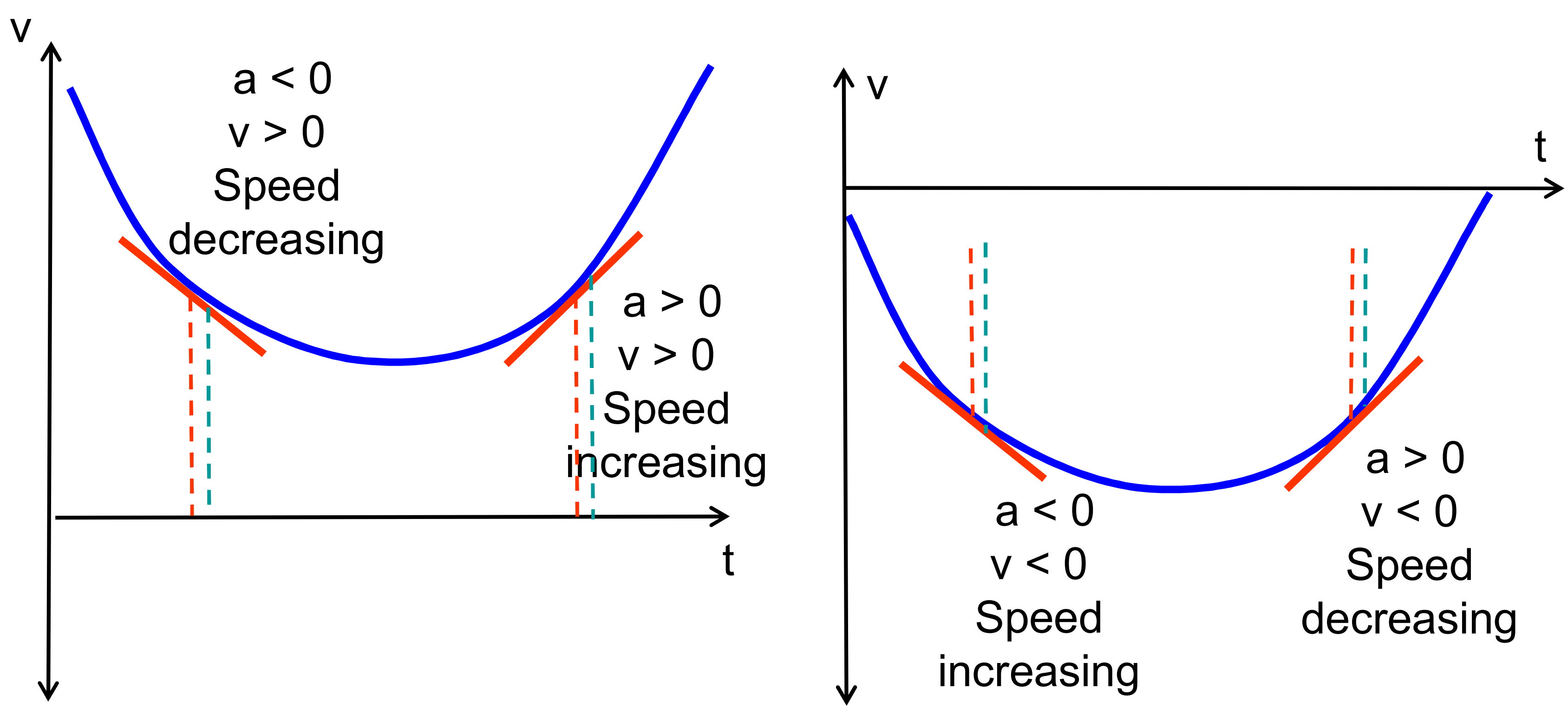
acceleration ≡ instantaneous acceleration

Acceleration is the slope of the v-versus-t curve



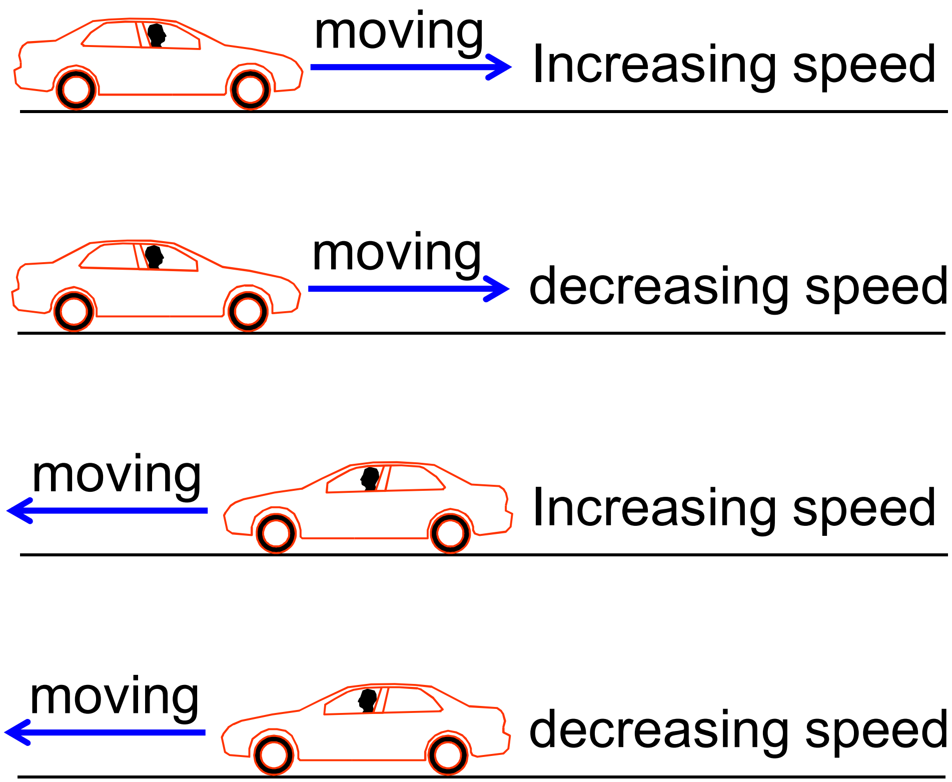
3-4 Acceleration
Acceleration direction

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases, if the signs are opposite, the speed decreases.



3-4 Acceleration
Checkpoint

What is the sign of acceleration?



Solution

Positive a →

Negative a ←

Negative a ←

Positive a →

3-4 Acceleration
Example

The position of a particle moving on the x axis is given by

$$x = 1.0 + 5.0\,t - 3.0\,t^3$$

with x in meters and t in seconds.

Find the acceleration of the particle as a function of time.

Solution

$$v = \frac{dx}{dt} = \frac{d}{dt}(1.0 + 5.0\,t - 3.0\,t^3) = 5.0 - (3)(3.0)\,t^2$$

$$= 5.0 - 9.0\,t^2$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(5.0 - 9.0\,t^2) = -(2)(9.0)\,t$$

$$= -18\,t$$

3-4 Acceleration

Question

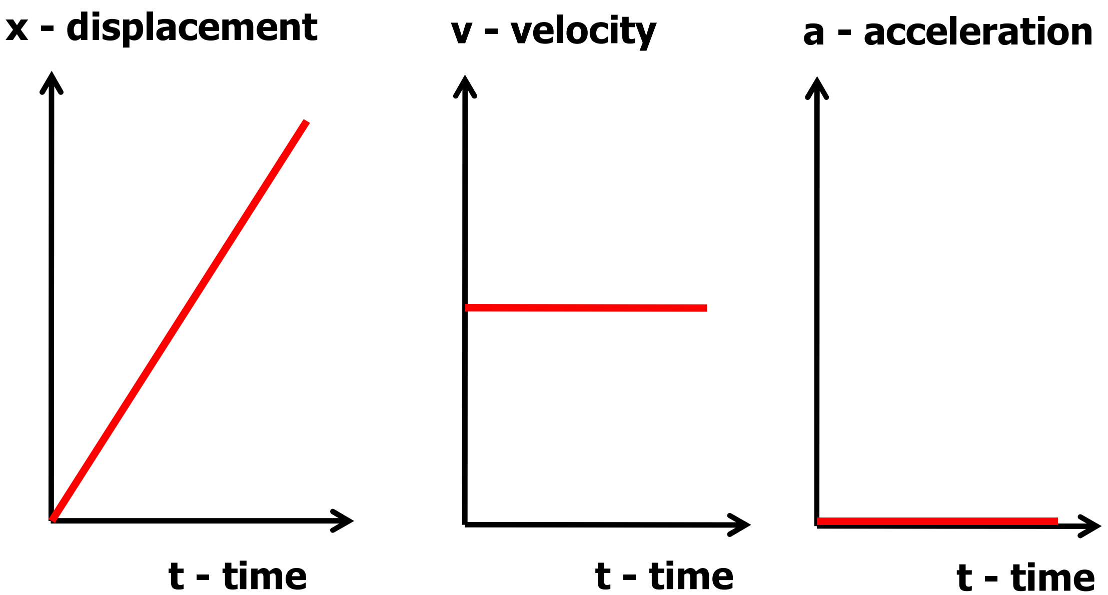
The position of a particle moving on the x axis is given by $\mathbf{x = 3t^2 - 5t + 20}$ with x in meters and t in seconds.

A. Find the position of the particle at t = 2 s?
B. Find the velocity of the particle at t = 2 s?
C. Find the acceleration of the particle as a function of time.
D. Find the velocity at t = 4 s.

Solution

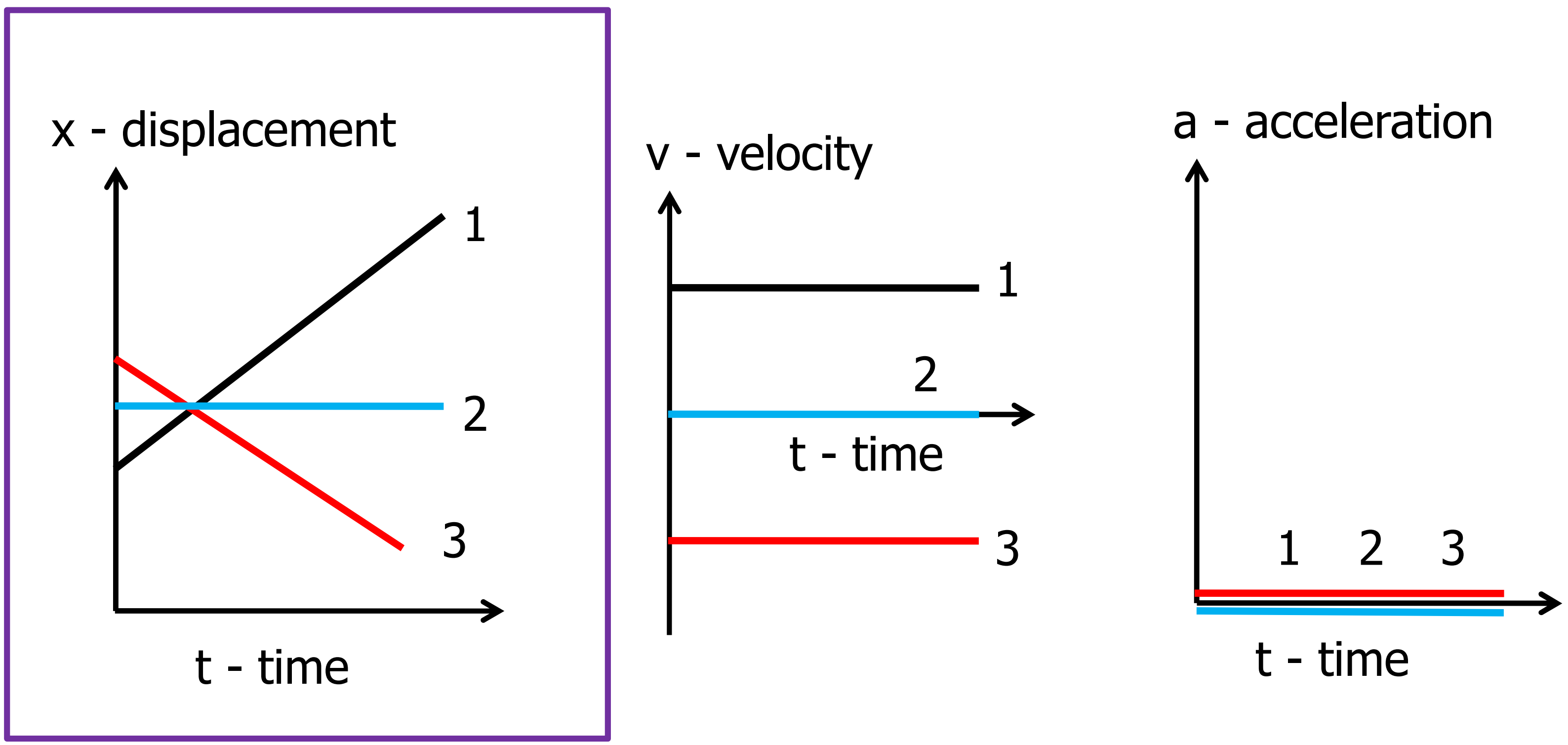
3-4 Acceleration
Motion Diagrams – Uniform Velocity

- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

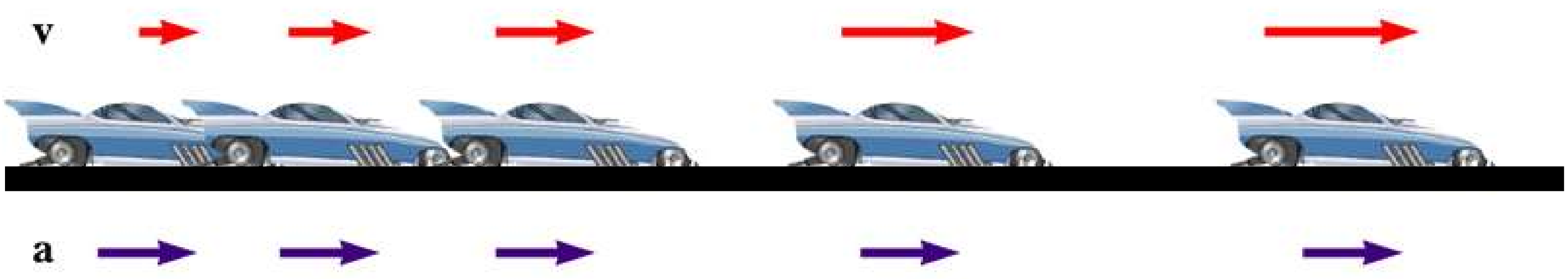


3-4 Acceleration
Example

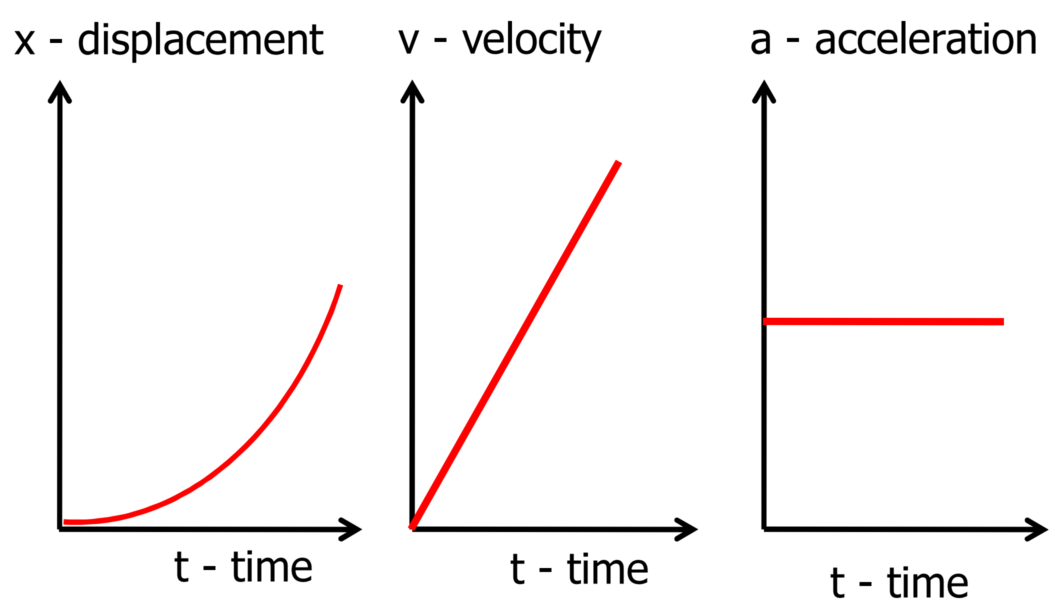
Plot the velocity – time and acceleration – time graphs for the given displacement – time graph



3-4 Acceleration
Motion Diagrams – Speeding Up

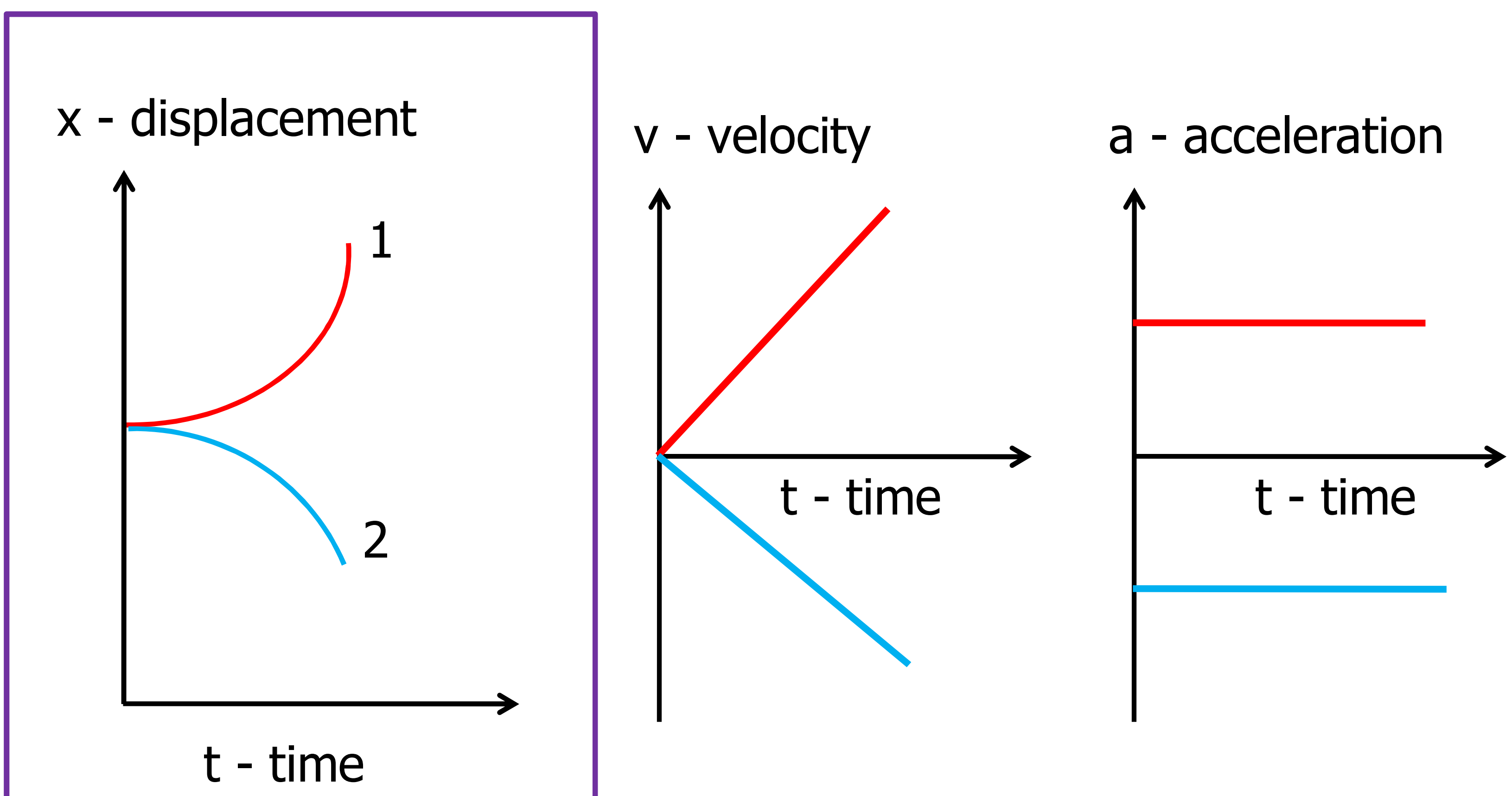


- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)



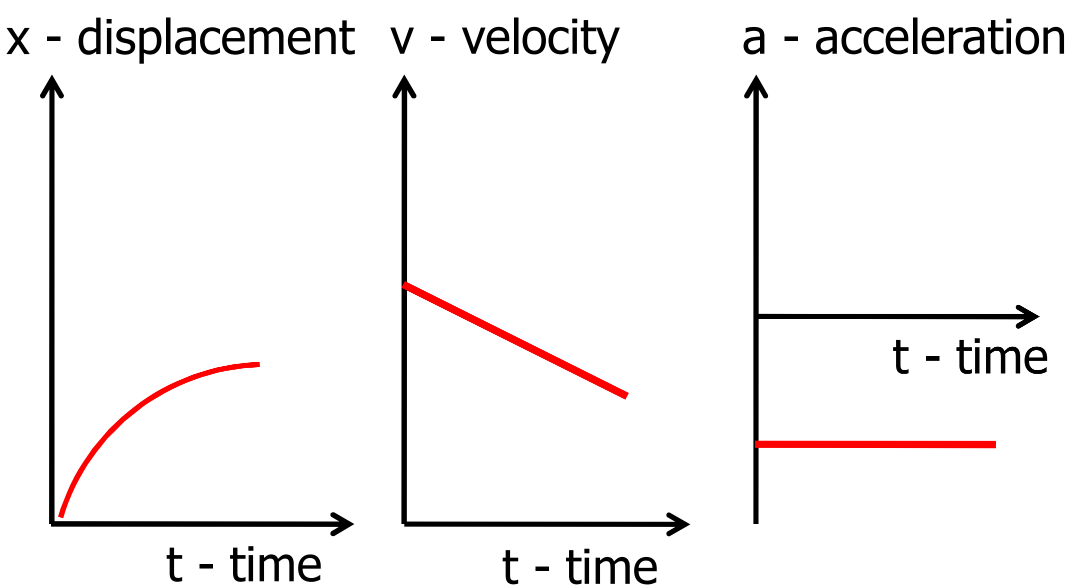
3-4 Acceleration
Example

Plot the velocity – time and acceleration – time graphs for the given displacement – time graph



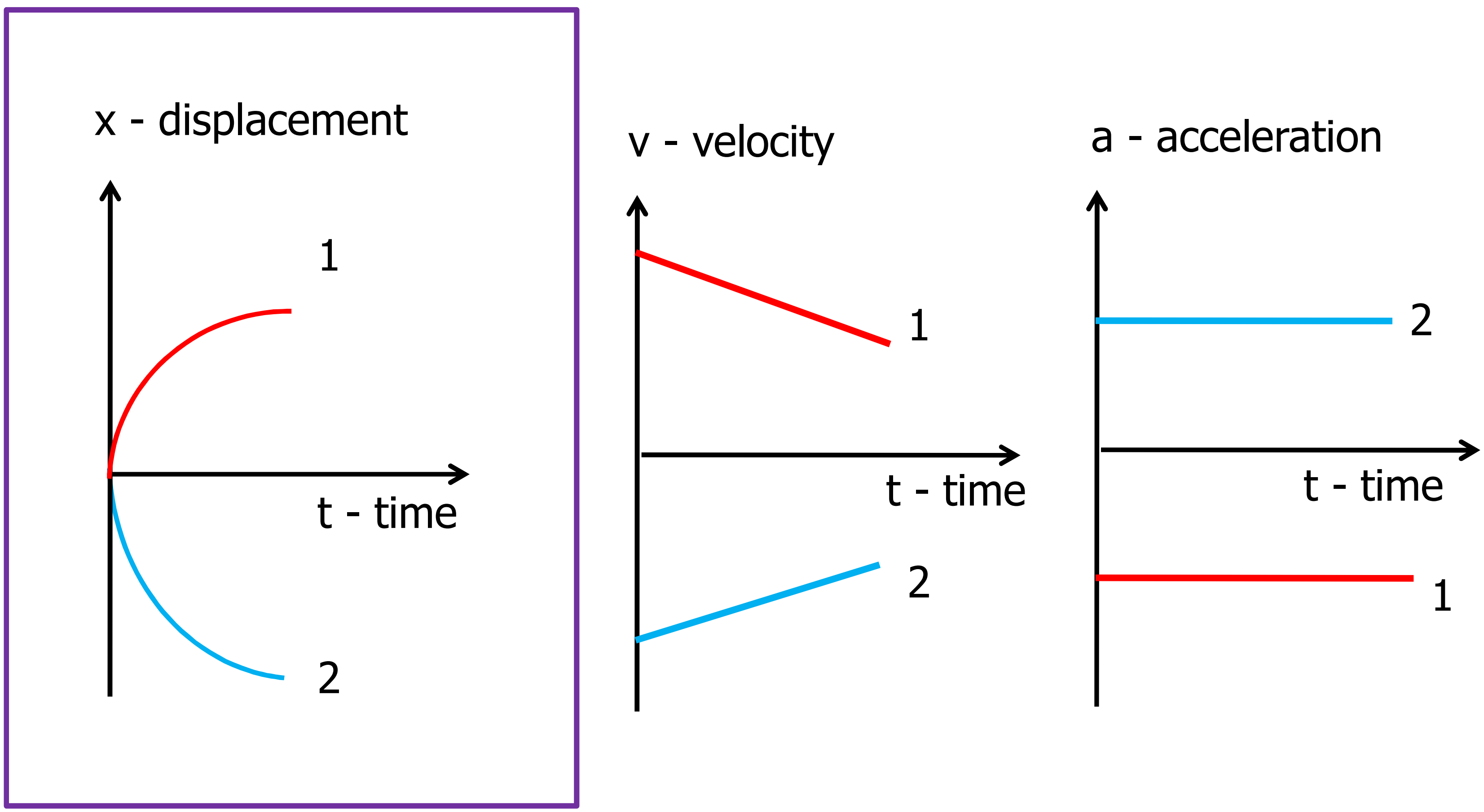
3-4 Acceleration
Motion Diagrams – Slowing Down

- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)



3-4 Acceleration
Example

Plot the velocity – time and acceleration – time graphs for the given displacement – time graph



3-5 Constant Acceleration
Question

The table shows the changes in the velocity of a moving object with respect to time.
a. Plot the velocity - time graph
b. Plot the acceleration - time graph

Time(s)	Velocity (m/s)
0	50
1	40
2	30
3	20
4	30
5	40
6	40

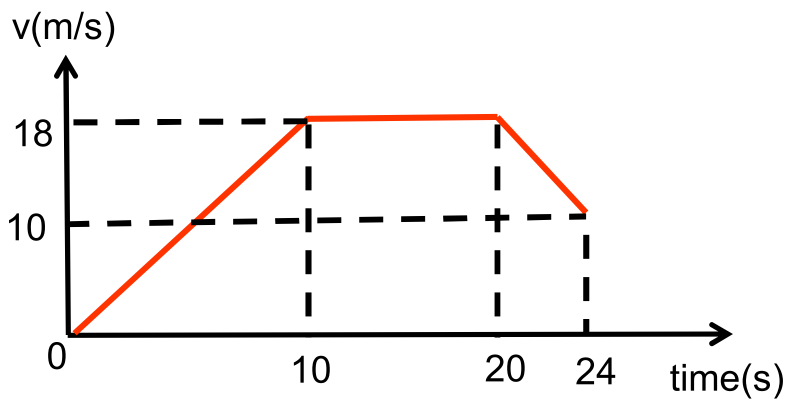
Solution

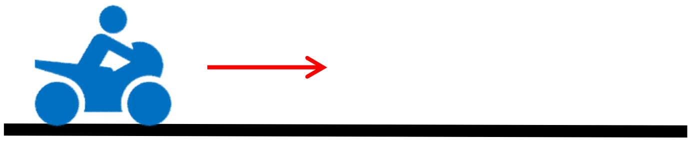
velocity - time graph

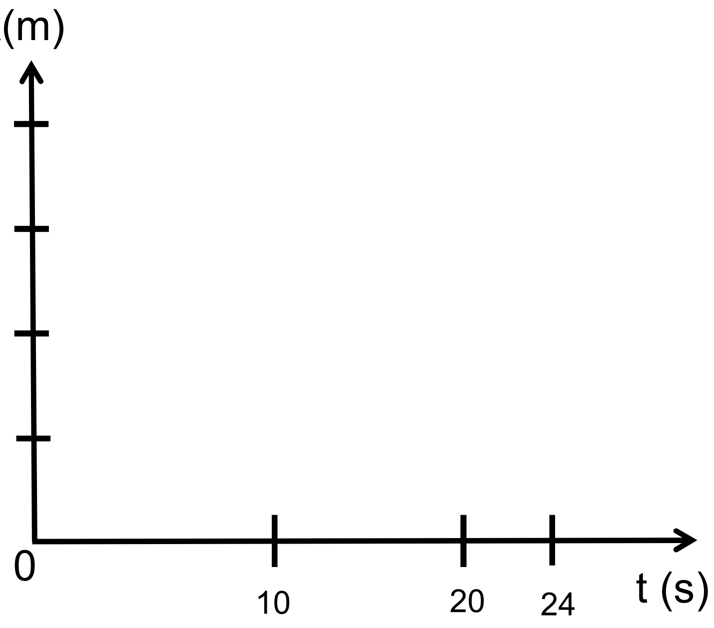
acceleration - time graph

3-4 Acceleration
Example

The graph shows the change in velocity of a motorbike with time.
Plot the “position- time” graph of this motion.



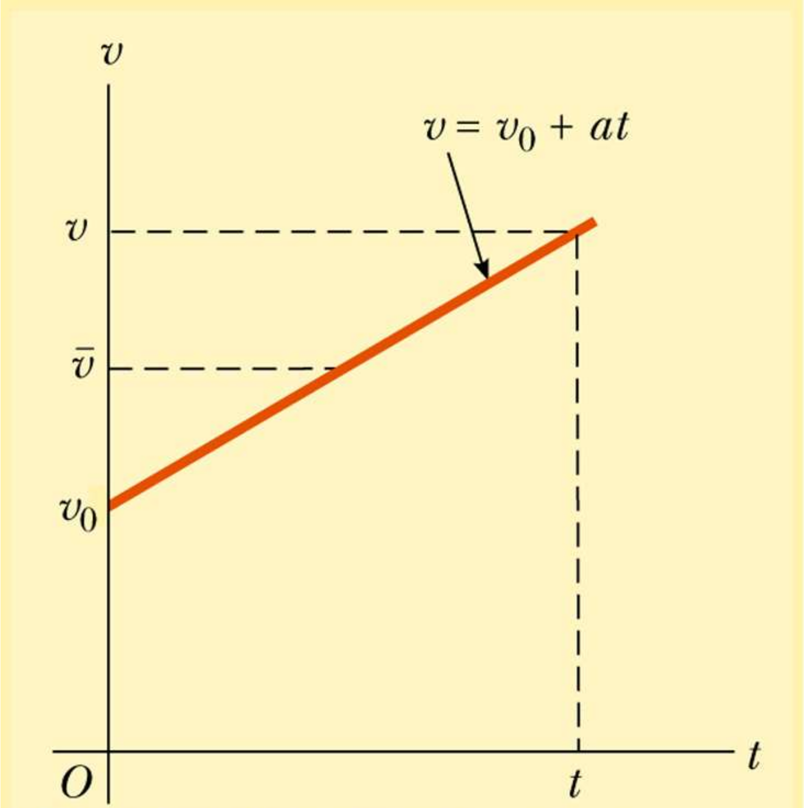




3-4 Acceleration

One-dimensional Motion with Constant Acceleration

If acceleration is uniform :



for constant *a*

$$v_{avarage} = \frac{v_0 + v_f}{2}$$

$$a = \frac{v_f - v_o}{t_f - t_0} = \frac{v_f - v_o}{t}$$

thus:

$$v_f = v_o + at$$

- This graph shows velocity as a function of acceleration and time.

3-5 Constant Acceleration
Formulas

Equations for motion with constant acceleration

Valid only for a = constant

Basic equations:

$$v_f = v_o + at$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

Useful and can be derived from the two basic equations:

$$v_f^2 = v_o^2 + 2a (x_f - x_0)$$

$$x_f - x_0 = \frac{1}{2}(v_f + v_0)t$$

$$x_f - x_0 = vt - \frac{1}{2}at^2$$

$$v_{avg} = \frac{1}{2} (v+v_0)$$

Notations	
t_i	→ 0
t_f	→ t
x_i	→ x_0
x_f	→ x
v_i	→ v_0
v_f	→ v

3-5 Constant Acceleration
Derivations

$$v = v_0 + a\,t$$

$$a = \frac{dv}{dt} \rightarrow dv = a\,dt$$

$$\int_{v_0}^{v} dv = \int_0^t a\,dt$$

constant

$$v - v_0 = \int_0^t a\,dt$$

$$v - v_i = a\int_0^t dt = a\,t$$

$$v = v_0 + a\,t$$

$$x - x_0 = v_0\,t + \frac{1}{2}a\,t^2$$

$$v = \frac{dx}{dt} \rightarrow dx = v\,dt$$

$$\int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$x - x_0 = \int_0^t v\,dt$$

$$x - x_0 = \int_0^t (v_0 + a\,t)\,dt$$

constants

$$x - x_0 = \int_0^t v_0 dt + \int_0^t a\,t\,dt$$

$$x - x_0 = v_0\int_0^t dt + a\int_0^t t\,dt$$

$$x - x_0 = v_0\,t + \frac{1}{2}a\,t^2$$

3-5 Constant Acceleration
Derivations

$$v = v_0 + a\,t$$

$$x - x_0 = v_0\,t + \frac{1}{2}a\,t^2$$

eliminate " t " →

How do we get *velocity equation* without "t"?

$$v^2 = v_0^2 + 2\,a\,(x - x_0)$$

$$v = v_0 + a\,t \;\rightarrow\; t = \frac{v - v_0}{a}$$

$$x - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

$$2\,a\,(x - x_0) = 2\,v_0(v - v_0) + (v - v_0)^2$$

$$2\,a\,(x - x_0) = 2\,v_0 v - 2\,v_0^2 + v^2 - 2\,v\,v_0 + v_0^2$$

$$2\,a\,(x - x_0) = v^2 - v_0^2$$

$$v^2 = v_0^2 + 2\,a\,(x - x_0)$$

3-5 Constant Acceleration

Checkpoint

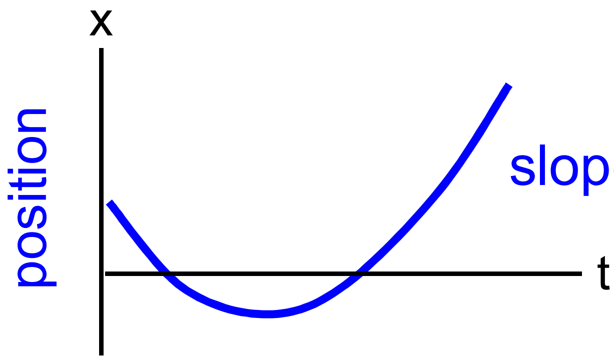
The following equations give the position of a particle. To which of these cases do the equations of this section apply?

Solution

$x = 5t + 2$	$a = 0$	Yes
$x = 2t^3 + 3t$	$a = 12t$	No
$x = \frac{2}{t^2}$	$a = \frac{12}{t^4}$	No
$x = 5t^2 + 4$	$a = 10$	Yes

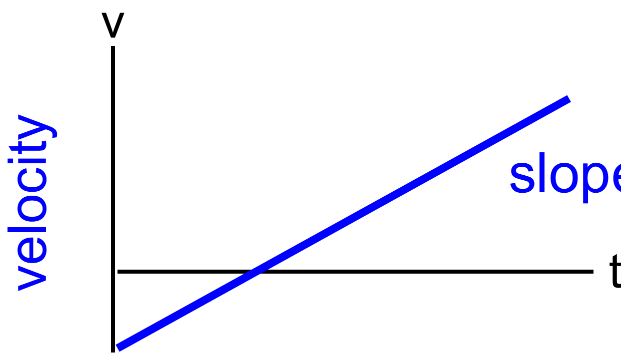
3-5 Constant Acceleration
Graphs

$$x - x_0 = v_0\, t + \frac{1}{2}\, a\, t^2$$



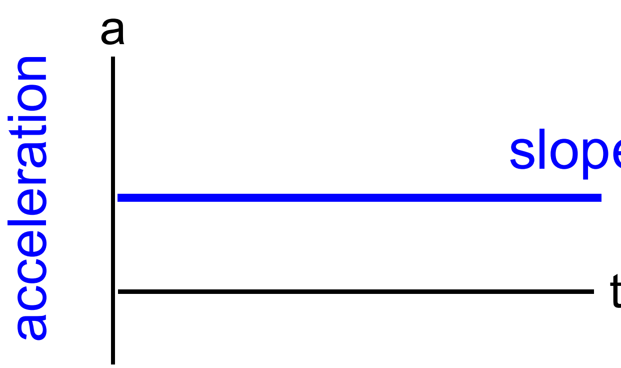
slope = $v_0 + a\, t = v$

$$v = v_0 + a\, t$$



slope = a = constant

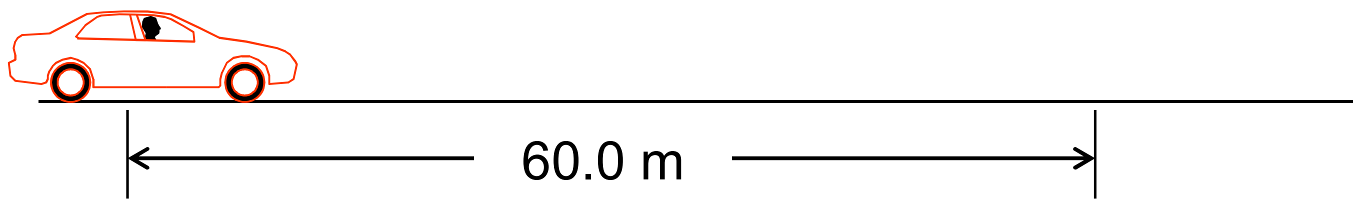
$$a = \text{constant}$$



slope = 0

3-5 Constant Acceleration
Example

Braking at a constant acceleration.



What is the acceleration in m/s^2?

Solution

Since we know the displacement $x - x_0$, the initial velocity v_0, and the final velocity v, we can use the following equation to find a

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(40.0\ \text{km/h})^2 - (110.0\ \text{km/h})^2}{2(60.0\ \text{m})} = \frac{-10500\ (\text{km/h})^2}{2(.0600\ \text{km})}$$

$$= -8.75 \times 10^4\ \text{km/h}^2$$

$$\equiv (8.75 \times 10^4\ \frac{\text{km}}{\text{h}^2})(\frac{1000\ \text{m}}{1\ \text{km}})(\frac{1\ \text{h}}{3600\ \text{s}})^2$$

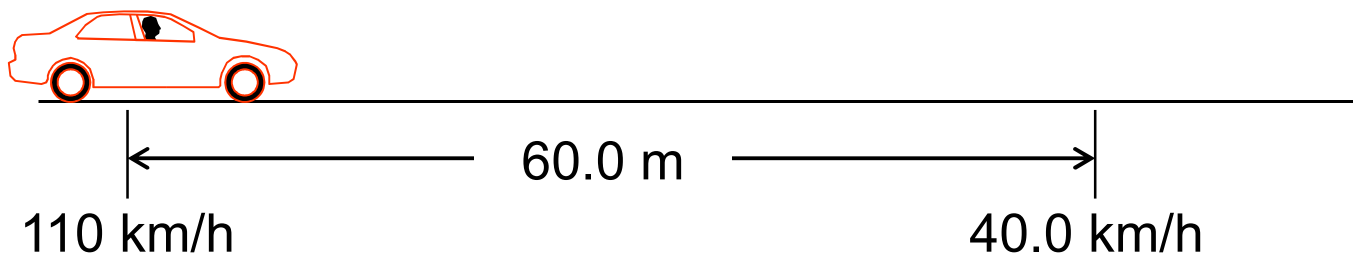
$$= 6.75\ \text{m/s}^2$$

$$a = -6.75\ \text{m/s}^2$$

3-5 Constant Acceleration

Example

Braking at a constant acceleration.



What is the time in seconds?

Solution

Since we know the displacement $x - x_0$, the initial velocity v_0, and the final velocity v, we can use the following equation to find t

$$x - x_0 = \frac{1}{2}(v + v_0)\,t$$

$$t = \frac{2\,(x - x_0)}{(v + v_0)} = \frac{2\,(0.0600\text{ km})}{(110.0\text{ km/h}) + (40.0\text{ km/h})} = 8.00 \times 10^{-4}\text{ h}$$

$$\equiv (8.00 \times 10^{-4}\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)$$

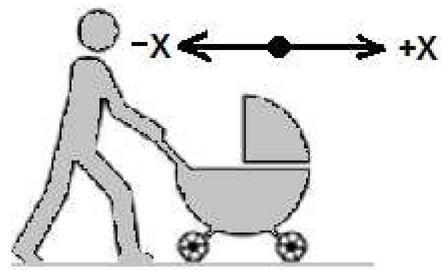
$$= 2.88\text{ s}$$

$$t = 2.88\text{ s}$$

3-5 Constant Acceleration
Example

A person pushing a stroller starts from rest, uniformly accelerating at a rate of 0.5 m/s^2. What is the velocity of the stroller after it has traveled 4.75 m?

Solution Choose right side as + and left side as -.
Since we know the displacement, the initial velocity v_0, and the acceleration, we can use the following equation to find final velocity.



$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x \quad \Rightarrow \quad v_f = \pm\sqrt{v_i^{\,2} + 2a\Delta x}$$

Put the numbers into the equation and solve for final velocity.

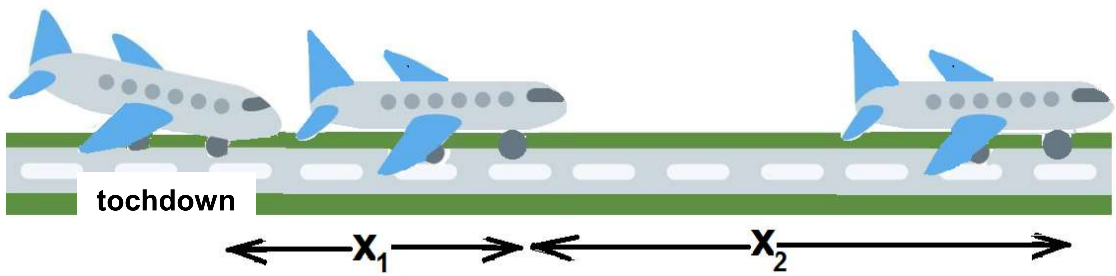
$$v_f = \pm\sqrt{(0\text{ m/s})^2 + 2(0.500\text{ m/s}^2)(4.75\text{ m})}$$

$$v_f = +2.18\text{ m/s}$$

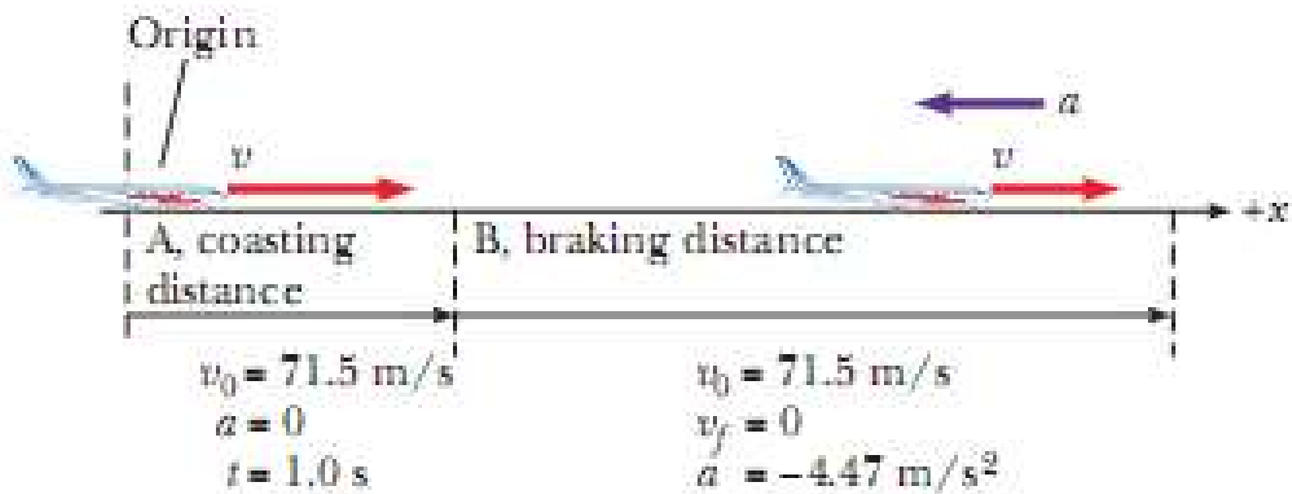
The stroller's velocity after accelerating for 4.75 m is 2.18 m/s to the right.

3-5 Constant Acceleration
Example

An airplane lands at a speed of 71.5 m/s and decelerates at the rate of – 4.47 m/s^2. If the plane travels at a constant speed of 71.5 m/s for 1 second after landing before applying the brakes, what is the total displacement of the airplane between tochdown on the runway and coming to rest?



Solution



3-5 Constant Acceleration
Example-Solution

Taking $a = 0$, $v_0 = 71.5$ m/s, and $t = 1.00$ s, find the displacement while the plane is coasting:

$$\Delta x_{\text{coasting}} = v_0 t + \tfrac{1}{2}at^2 = (71.5\text{ m/s})(1.00\text{ s}) + 0 = 71.5\text{ m}$$

Use the time-independent kinematic equation to find the displacement while the plane is braking.

$$v^2 = v_0^2 + 2a\,\Delta x_{\text{braking}}$$

Take $a = -4.47\text{ m/s}^2$ and $v_0 = 71.5$ m/s. The negative sign on a means that the plane is slowing down.

$$\Delta x_{\text{braking}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (71.5\text{ m/s})^2}{2.00(-4.47\text{ m/s}^2)} = 572\text{ m}$$

Sum the two results to find the total displacement:

$$\Delta x_{\text{coasting}} + \Delta x_{\text{braking}} = 72\text{ m} + 572\text{ m} = 644\text{ m}$$

3-5 Constant Acceleration
Question

1. Which of the following statements is true of acceleration?

   A. Acceleration always has the same sign as displacement.

   B. Acceleration always has the same sign as velocity.

   C. The sign of acceleration depends on both the direction of motion and how the velocity is changing.

   D. Acceleration always has a positive sign.

2. A ball initially at rest rolls down a hill and has an acceleration of 3.3 m/s^2. If it accelerates for 7.5 s, how far will it move during this time?

   A. 12 m B. 93 m C. 120 m D. 190 m

3-5 Constant Acceleration
Question

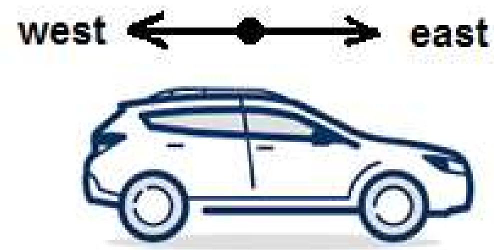
A car moving eastward along a straight road increases its speed uniformly from 16 m/s to 32 m/s in 10.0 s.

a. What is the car's average acceleration?

b. What is the car's average velocity?

c. How far did the car move while accelerating?

Show all of your work for these calculations.



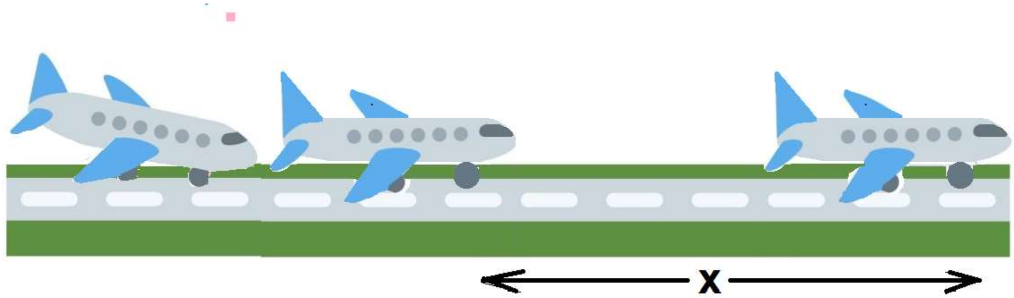
Solution

Answers: a. 1.6 m/s² eastward
b. 24 m/s
c. 240 m

3-5 Constant Acceleration

Question

A airplane lands with 80 m/s, applying brakes 2 seconds after landing. Calculate the acceleration to stop the airplane within 500 m.



Solution

Answer $a = -9.41\ \mathrm{m/s^2}$

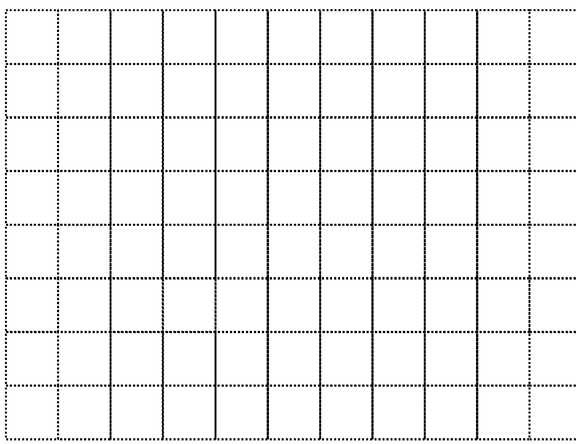
3-5 Constant Acceleration
Question

The table shows the changes in the velocity of a moving object with respect to time.
a. Plot the velocity - time graph
b. Plot the acceleration - time graph

Time(s)	Velocity (m/s)
0	50
1	40
2	30
3	20
4	20
5	5

Solution

velocity - time graph



acceleration - time graph