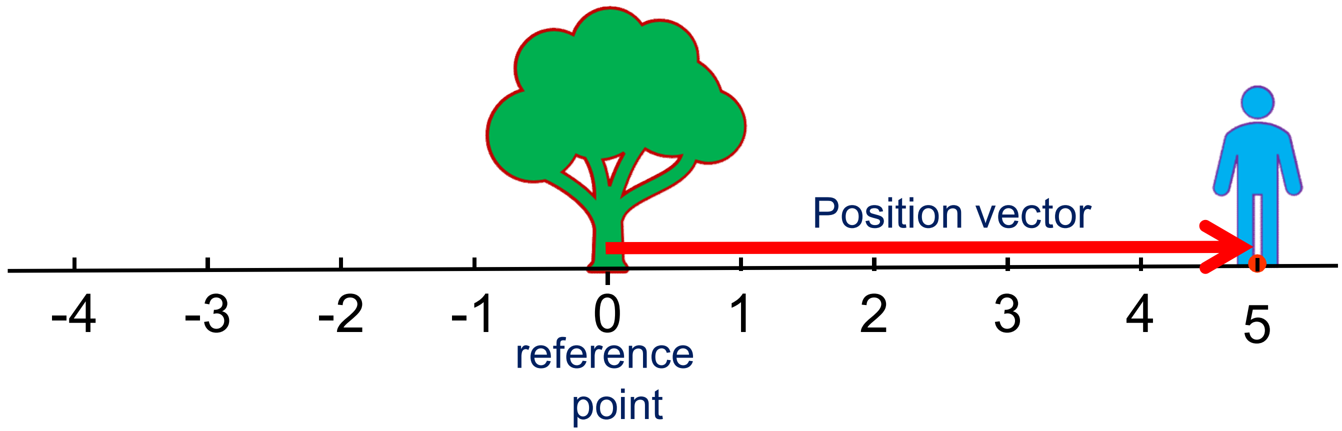
Chapter 3 - Part A
Motion Along a Straight Line

3-1 Position and Displacement
3-2 Average Velocity and Average Speed
3-3 Instantaneous Velocity and Speed

3-1 Position and Displacement
Position

Position is defined in terms of a frame of reference.

► Position is the distance of an object to a reference point in a direction. So position is a vector quantity.

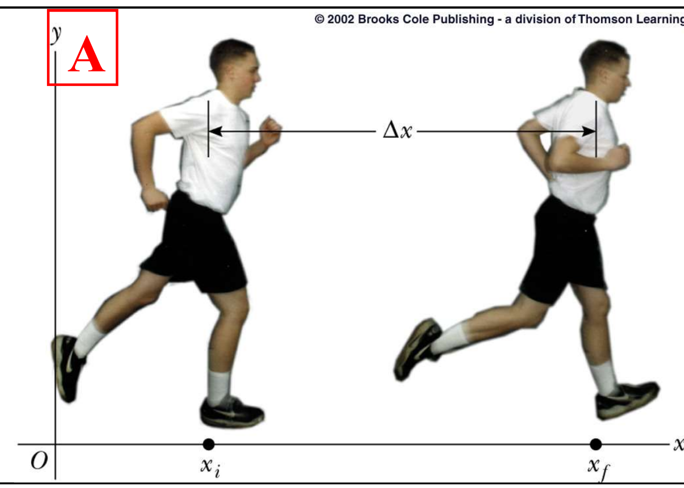


► Frame A:

$x_i > 0$ and $x_f > 0$

► Initial position : x_i

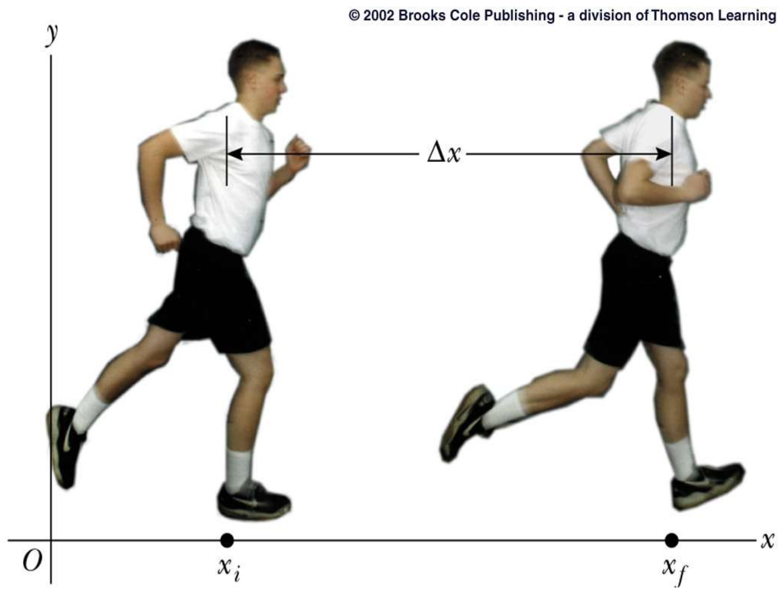
► Final position : x_f



3-1 Position and Displacement
Position

Displacement is the change in position of an object.

- It is a vector quantity (i.e. needs directional information)
- + or - is generally sufficient to indicate direction for one-dimensional motion



Position is represented as; $\Delta x = x_f - x_i$

displacement = final position – initial position

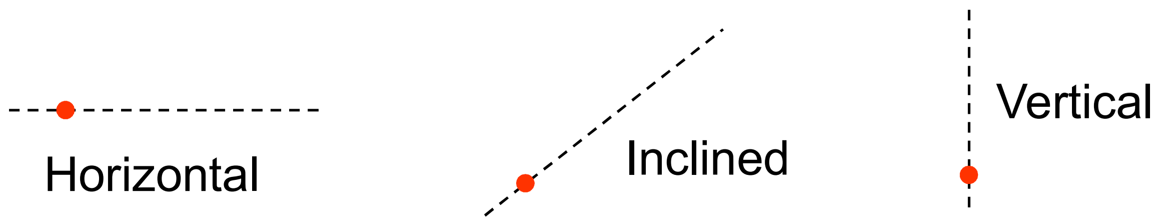
3-1 Position and Displacement
Position

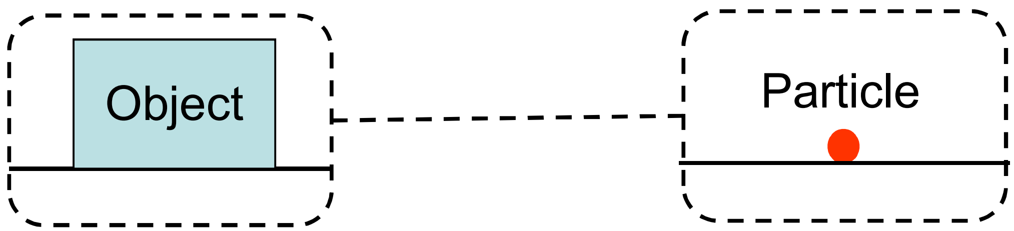
Distance	Displacement
The actual length of the path covered by a certain interval of time is called travelled by the body.	The shortest distance covered by a body in a particular direction while moving from one point to another is called its displacement.
It is a scalar quantity.	It is a vector quantity.
Distance is always positive.	Displacement can be both positive or negative.
Distance travelled by a moving body in a certain interval of time can never be zero.	Displacement of a moving body in a certain interval of time can be zero.
Distance ≥ Displacement	Displacement ≤ Distance

3-1 Position and Displacement

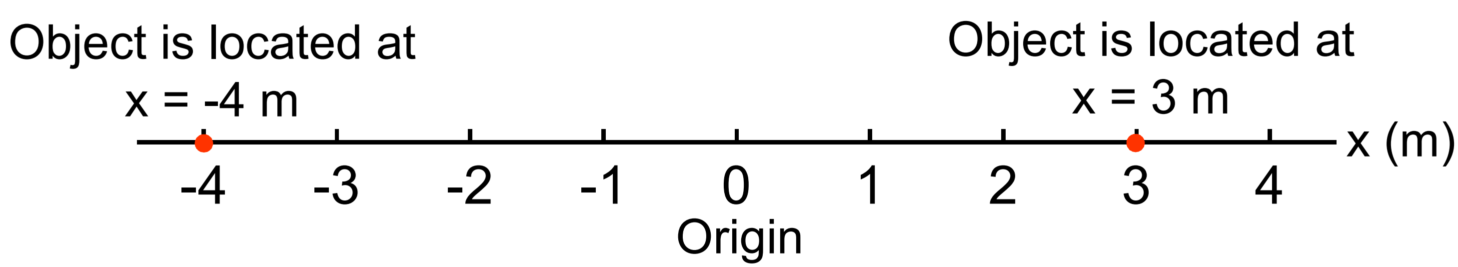
Position

Motion along a straight line ≡ One-dimensional motion





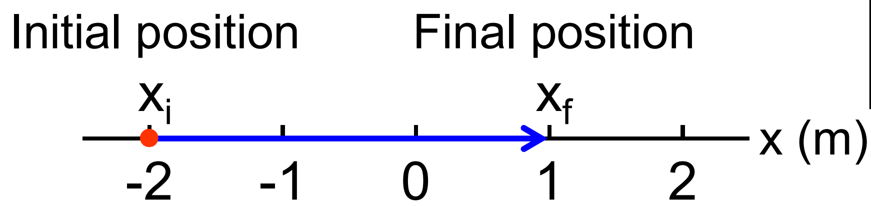
An object moves like a particle



3-1 Position and Displacement
Displacement

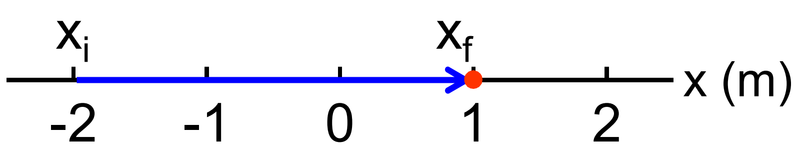
Displacement = final position - initial position

$\Delta x = x_f - x_i$

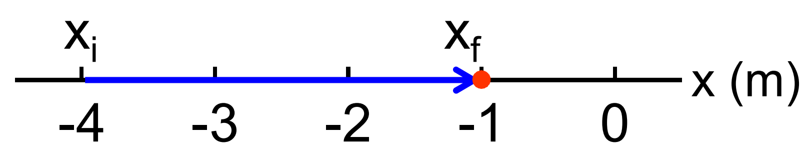


Δ = Greek letter, delta
$\Delta \equiv$ change
$\Delta x \equiv$ change in x

Displacement in the positive direction is positive.

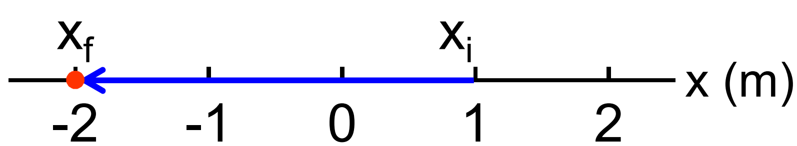


$\Delta x = 1 - (-2) = 3$ m

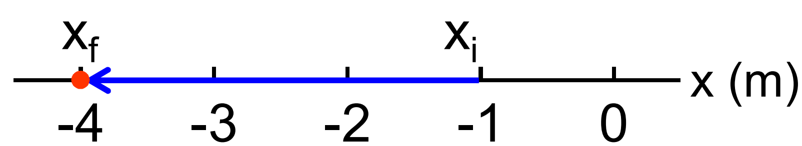


$\Delta x = -1 - (-4) = 3$ m

Displacement in the negative direction is negative.

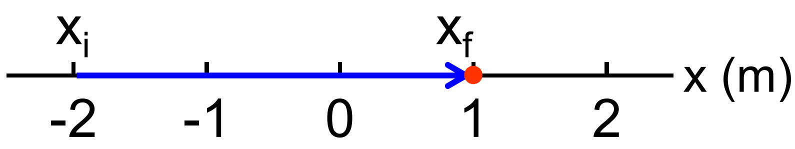


$\Delta x = -2 - 1 = -3$ m



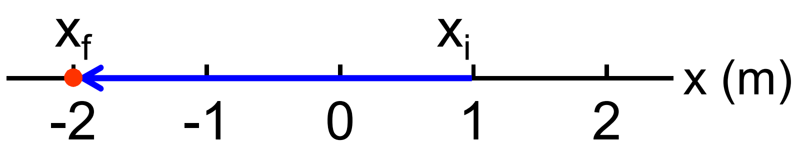
$\Delta x = -4 - (-1) = -3$ m

3-1 Position and Displacement
Displacement is a vector quantity



$\Delta x = 3$ m

Displacement $\Delta x = 3$ m means the object position has changed by 3 m in the positive direction.



$\Delta x = -3$ m

Displacement $\Delta x = -\ 3$ m means the object position has changed by 3 m in the negative direction.

To determine the displacement of an object, you need to specify

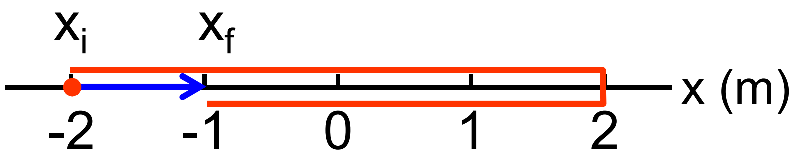
1- Magnitude (The distance between the initial and final positions. Always positive)

2- Direction (Negative or positive direction)

→ Displacement is a **vector quantity**.

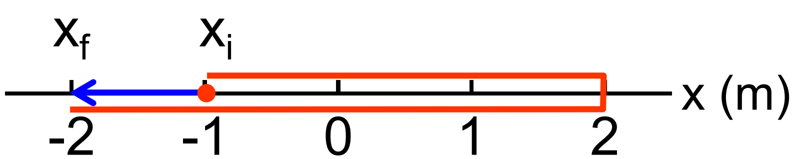
$$\Delta x = \underbrace{-}_{\text{Direction.}}\ \overbrace{3\text{ m}}^{\text{Magnitude (Absolute value). Always positive.}}$$

3-1 Position and Displacement
Total distance



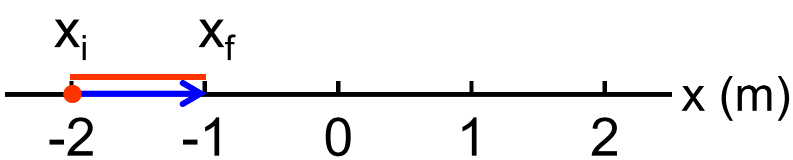
Δx = 1 m.
Total distance = 7 m.

Total distance is not always equal to the magnitude of displacement.



Δx = - 1 m.
Total distance = 7 m.

Total distance is always positive and does not depend on the direction.
→ Total distance is a **scalar quantity**.



Δx = 1 m.
Total distance = 1 m.

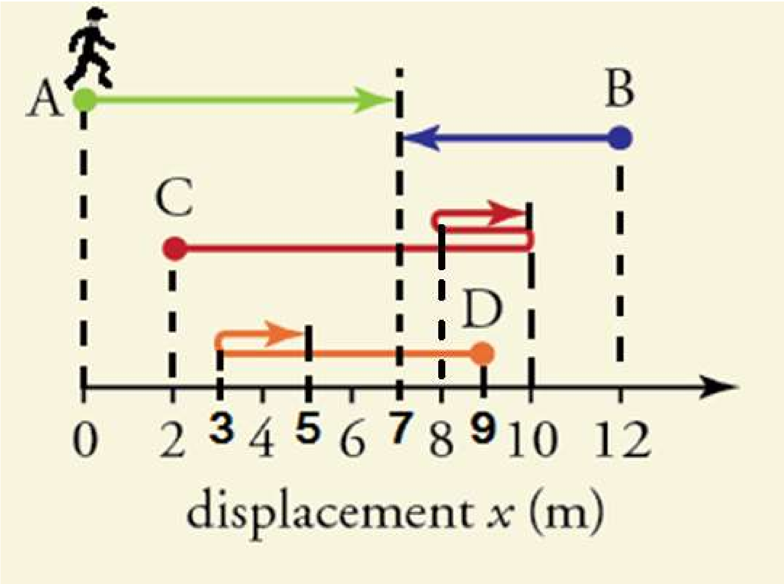
Displacement depends only on the initial and final positions.
Total distance depends on the path.

3-1 Position and Displacement
Example

Find the followings for the paths A, B, C and D in the figure below:

a) The distance traveled.

b) The displacement from start to finish.



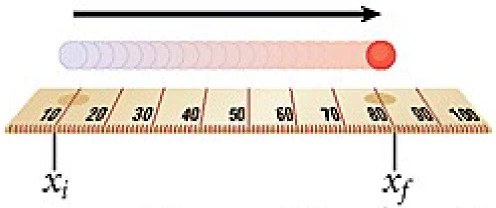
Solution:

3-1 Position and Displacement
Checkpoint

Look at the pictures and find the displacement of the ball in each case.

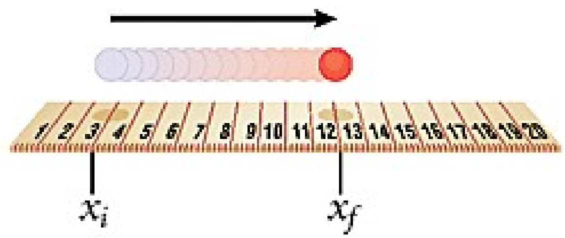
Positive

a)



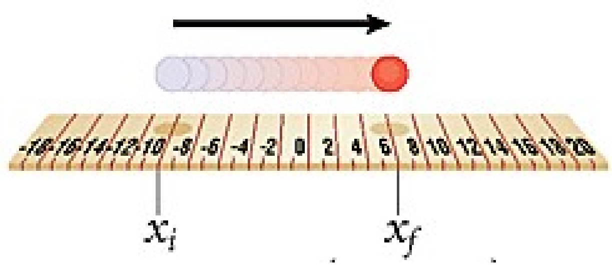
$\Delta x = x_f - x_i = 80\text{ cm} - 10\text{ cm} = +70\text{ cm}$

b)



$\Delta x = x_f - x_i = 12\text{ cm} - 3\text{ cm} = +9\text{ cm}$

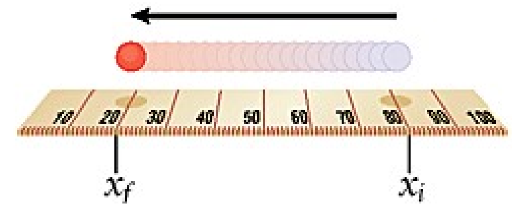
c)



$\Delta x = x_f - x_i = 6\text{ cm} - (-10\text{ cm}) = +16\text{ cm}$

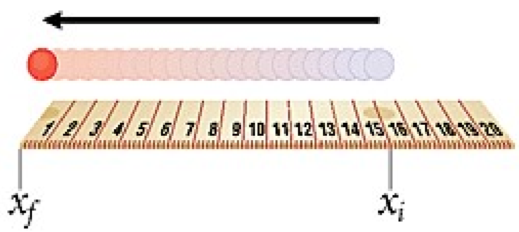
Negative

d)



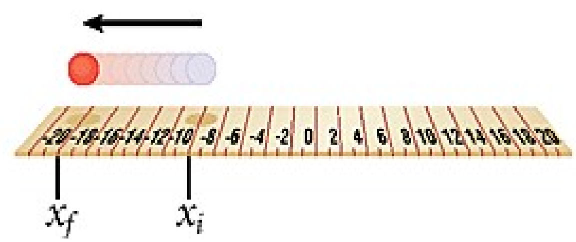
$\Delta x = x_f - x_i = 20\text{ cm} - 80\text{ cm} = -60\text{ cm}$

e)



$\Delta x = x_f - x_i = 0\text{ cm} - 15\text{ cm} = -15\text{ cm}$

f)



$\Delta x = x_f - x_i = -20\text{ cm} - (-10\text{ cm}) = -10\text{ cm}$

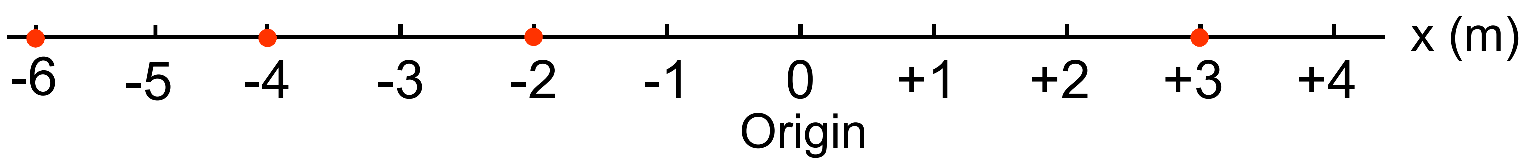
3-1 Position and Displacement
Checkpoint

What is the direction of the following displacements?

Initial position	Final position
-4 m	-2 m
-2 m	-6 m
3 m	-2 m

Solution

Direction
Positive
Negative
Negative

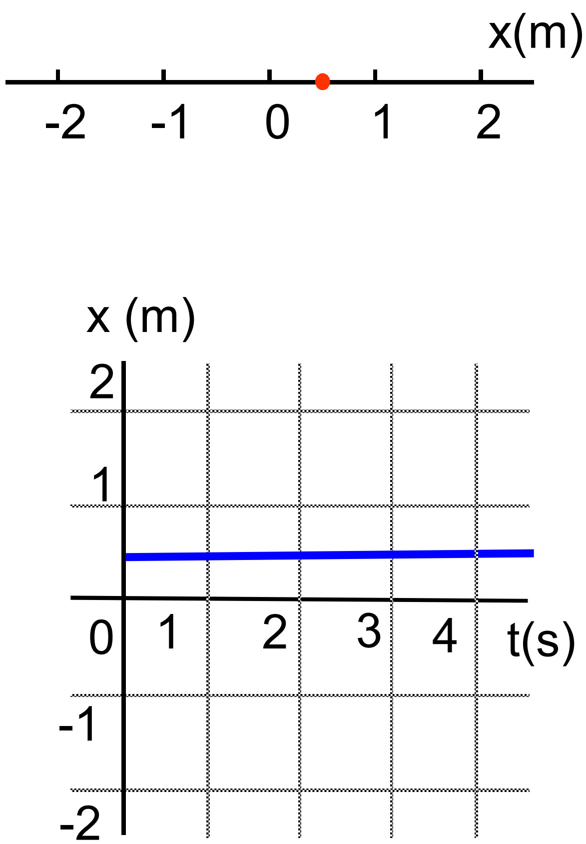


3-1 Position and Displacement

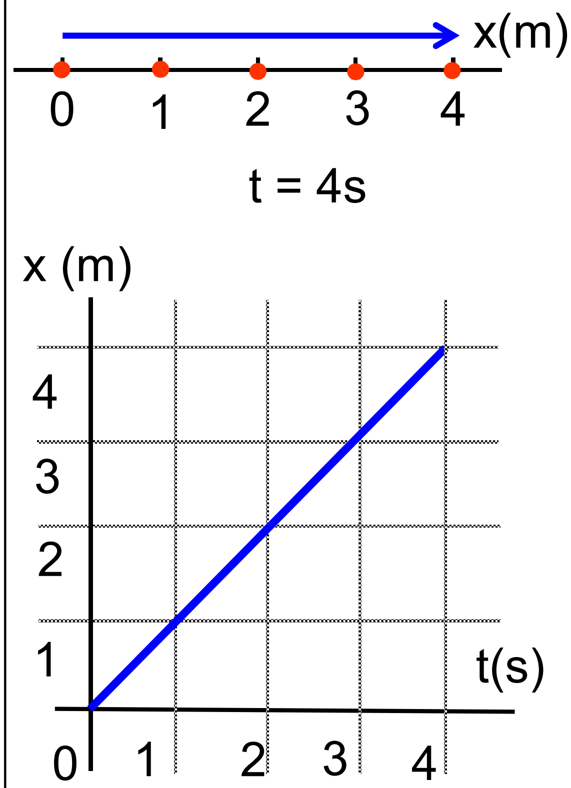
Position-time graph

Displacement is a vector quantity. A displacement-time graph is able to show if an object is going forwards or backwards.

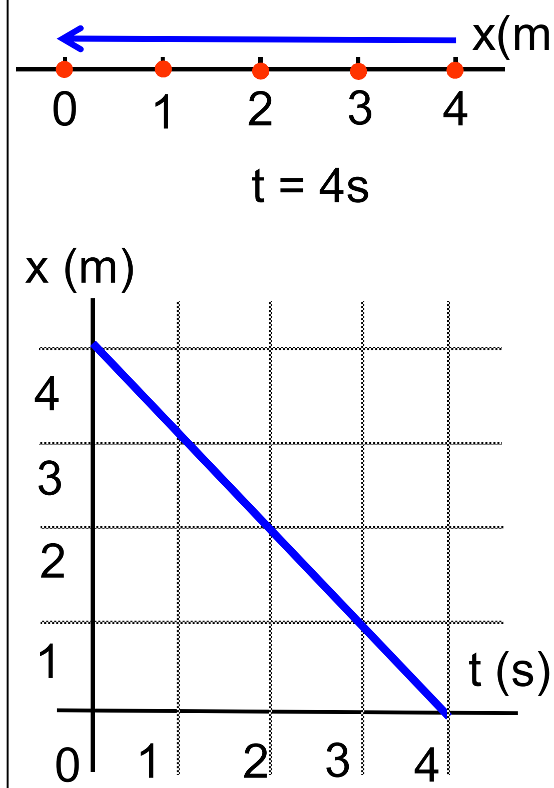
Stationary particle



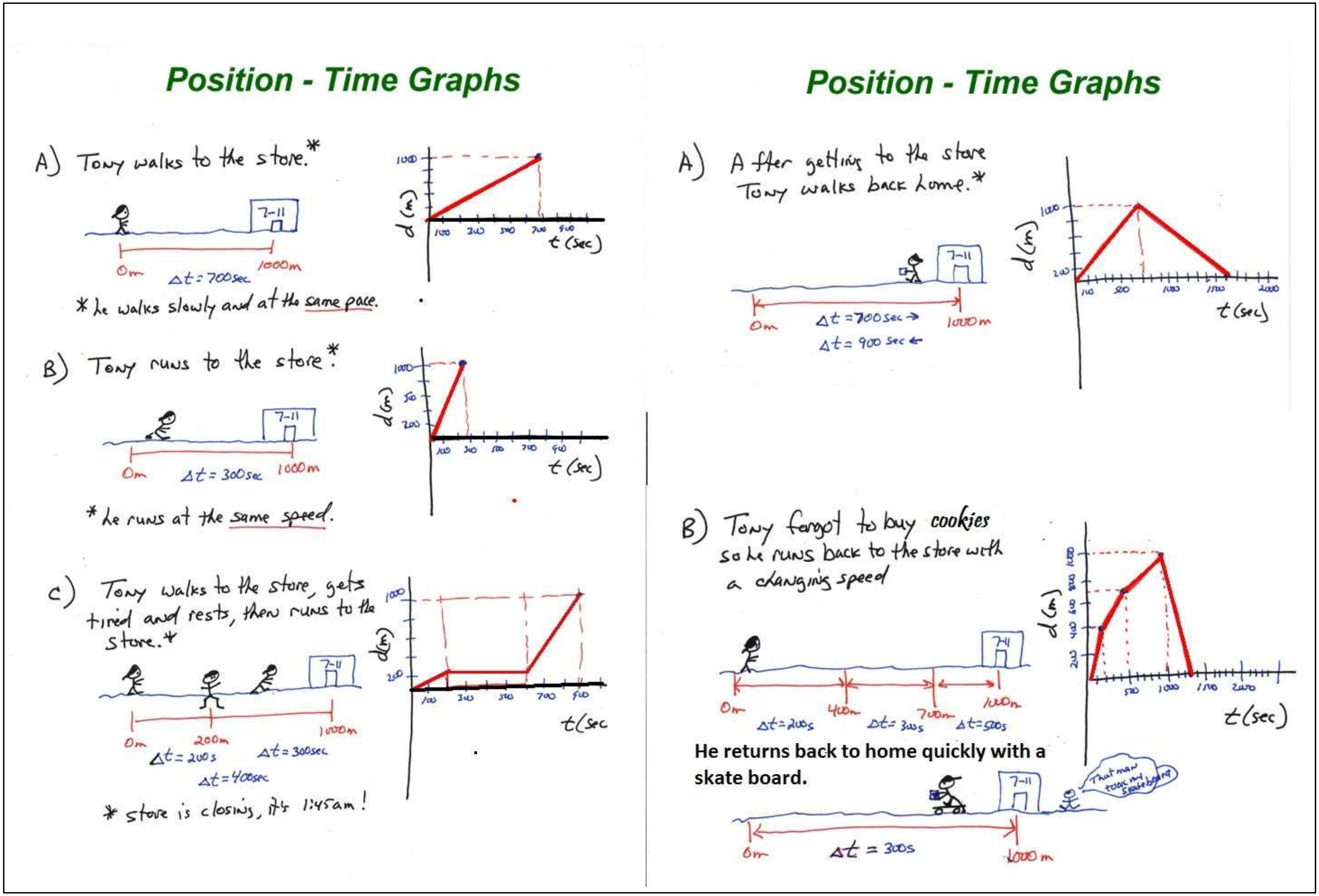
Particle moving forward from point zero



Particle moving backward to the point zero

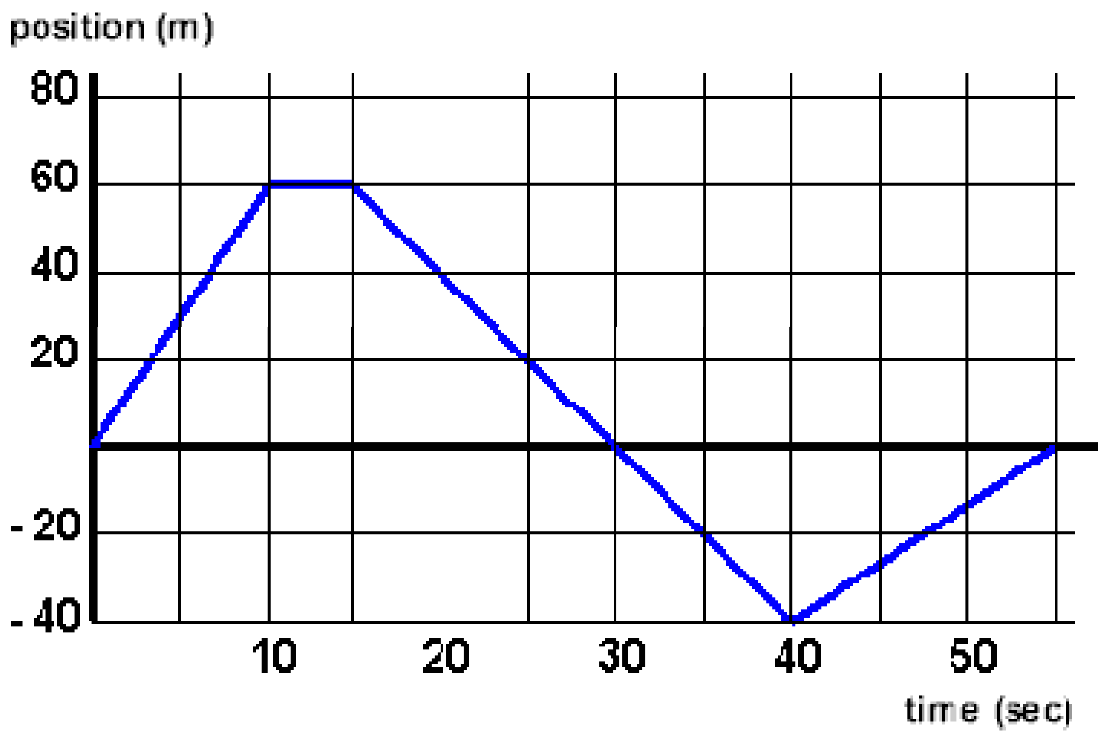


3-2 Average Velocity and Average Speed
Example: Drawing position – time graph



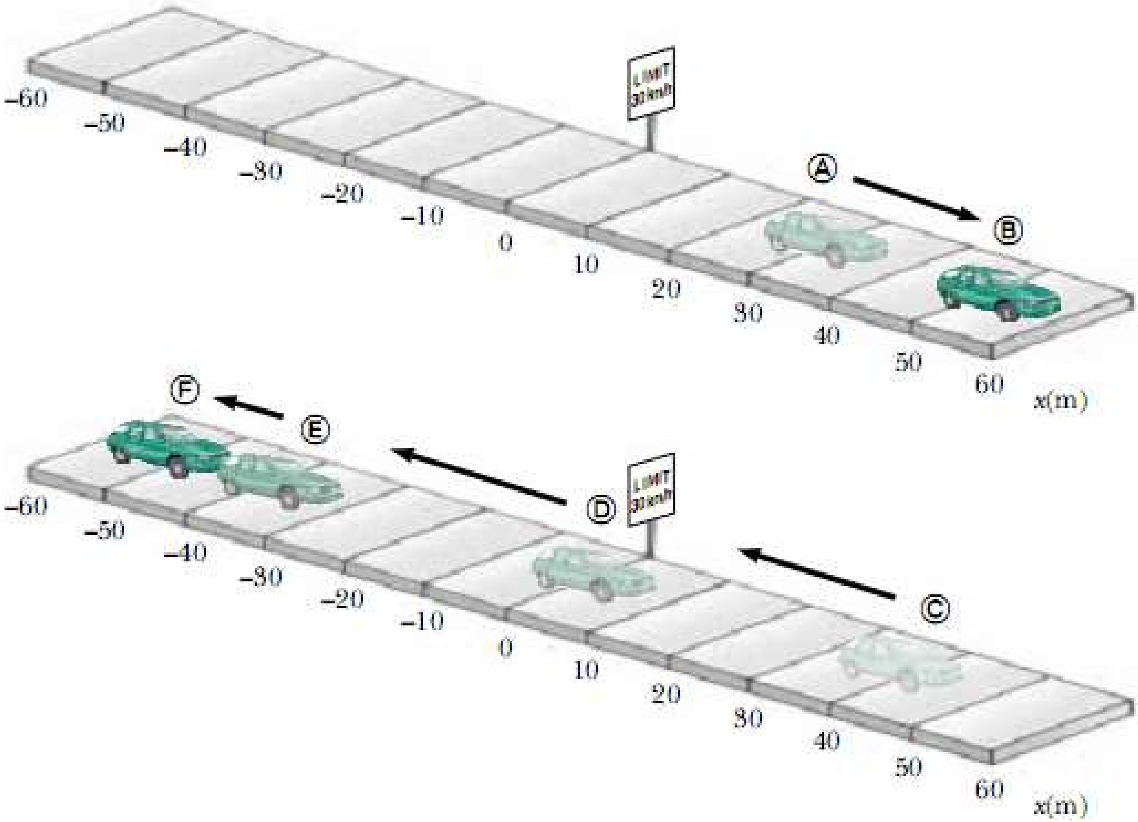
3-1 Position and Displacement
Example

An objects follows a path according to the graph below.
a. If an object goes back to where is started in certain time, then its displacement is zero. When did the object reach the point zero?
b. What is the direction of the motion between 0-10 seconds.
c. Where is the object stopping?
d. What is the position of the object at 25 s and 35 s.
e. What is total distance taken by the object during 55 s?
f. At what time intervals, the object moves in the positive direction?



3-1 Position and Displacement
Position-time graph

Draw the position-time graph of the car below.



Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

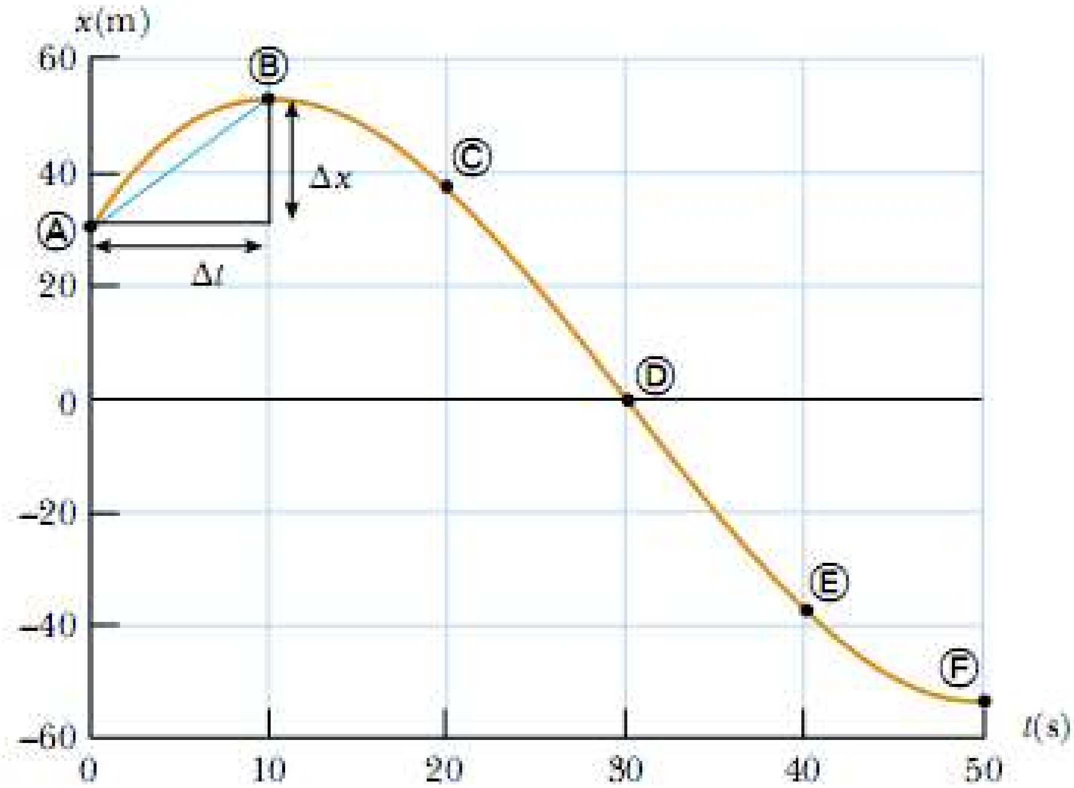
3-1 Position and Displacement
Position-time graph

Position-time graph is <u>not necessarily a straight line</u>, even though the motion is along x-direction.

The graph on the right shows the changes in position according to the position-time table.

Position of the Car at Various Times

Position	t(s)	x(m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	−37
F	50	−53

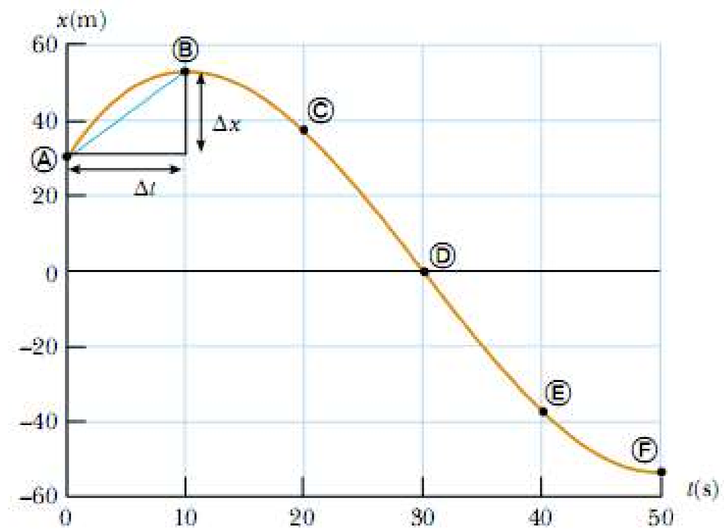


3-1 Position and Displacement

Position-time graph

Example

The graph shows the change in position of an object with respect to time. Answer the following questions using this graph.



a. What is the position of the object at point B?

b. What is the position of the object at point D?

c. What is the displacement of the object between the points A and B?

d. What is the displacement of the object between the points C and D?

e. What is the displacement of the object between the points B and F?

f. What is the displacement of the object between 0 and 30 s?

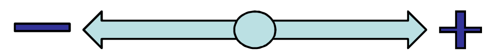
g. What is the displacement of the object between 10 s and 50 s?

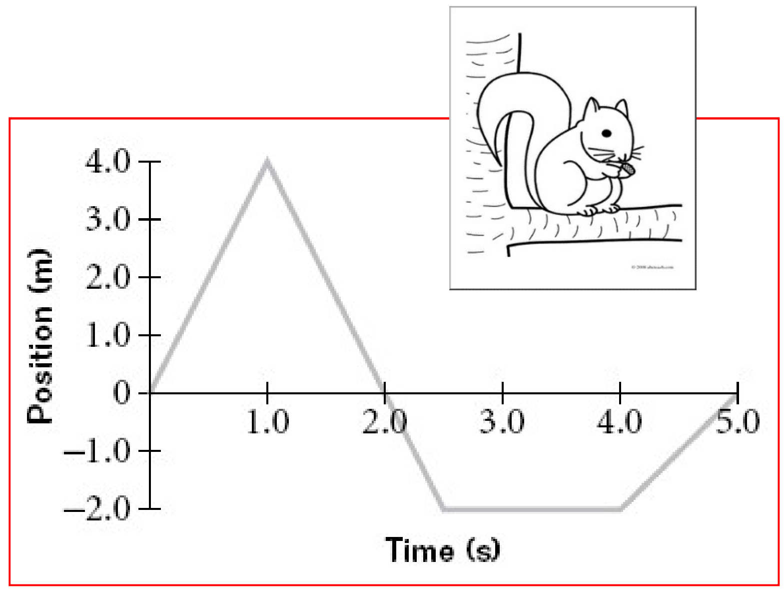
3-1 Position and Displacement

Position-time graph

Example

Use the position-time graph of a squirrel running back and forth along a branch of a tree to answer question.





1. What is the squirrel's displacement at time t = 3.0 s? (From 0 to 3 s)

   A. –6.0 m   B. –2.0

   C. +0.8 m   D. +2.0 m

2. What is the squirrel's displacement at time t = 1.0 s? (From 0 to 1 s)

   A. 0.0 m   B. –4.0 m

   C. +0.8 m   D. +4.0 m

3. What is the distance taken by the squirrel between 0 – 5.0 s?

   A. +4.0 m   B. – 1.0 m

   C. 0.0 m   D. +12.0 m

4. At what time intervals does the squirrel moves in negative direction?

   A. 0 – 1 s   B. 1 – 2 and 4 – 5 s

   C. 4 – 5 s   D. 0 - 1 and 2 – 2.5 s

3-2 Average Velocity and Average Speed

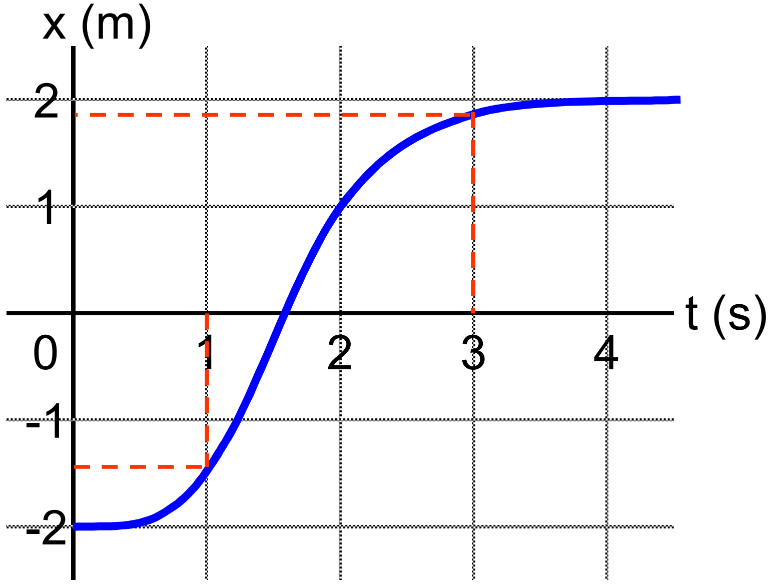
Average velocity

The average velocity v_{avg} for a time interval is:

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{\text{Final position - Initial position}}{\text{Final time - Initial time}}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Since Δt is always positive, average velocity has the same sign as the displacement. SI unit for average velocity is m/s.



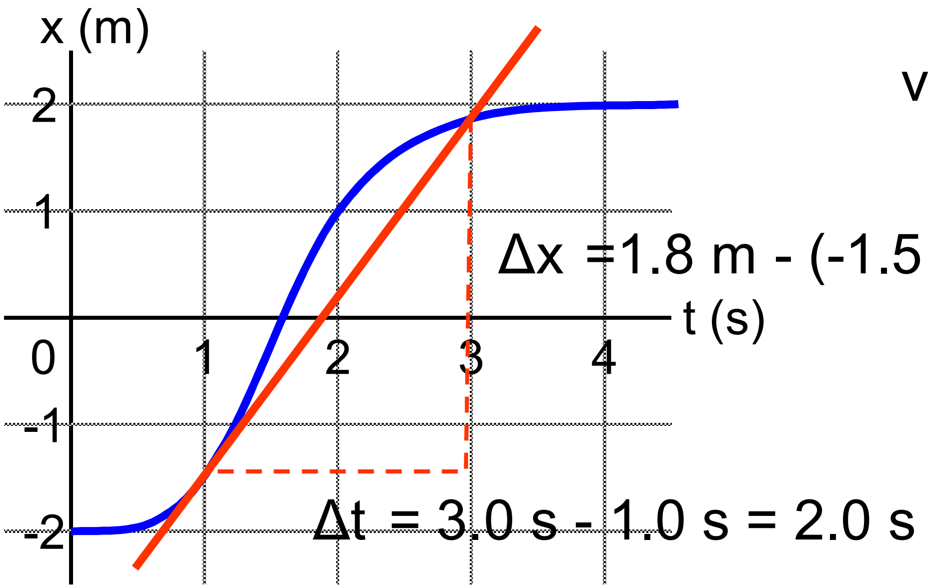
The average velocity for the time interval t = 1.0 s to t = 3.0 s is

$$v_{avg} = \frac{1.8 - (-1.5)}{3.0 - 1.0}\,\frac{m}{s} = \frac{3.3}{2.0}\,\frac{m}{s}$$

$$= 1.7\,\frac{m}{s}$$

3-2 Average Velocity and Average Speed

Average velocity from x-t graph



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

On the x-t graph, the average velocity v_{avg} for a time interval is the **slope of the straight line connecting the initial and final points**.

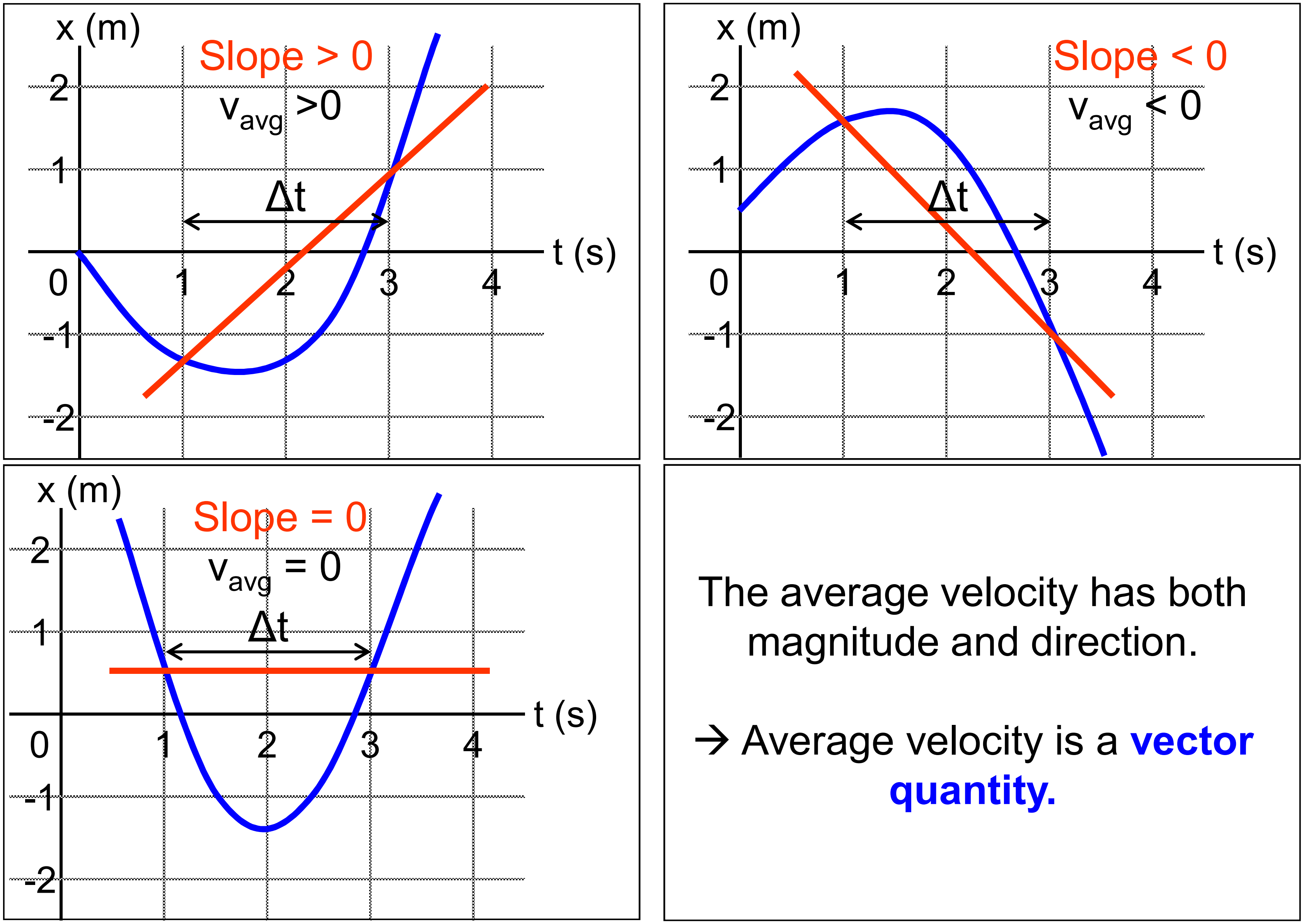
The average velocity for the time interval t = 1.0 s to t = 3.0 s is

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{3.3}{2.0}\frac{\mathrm{m}}{\mathrm{s}} = 1.7\,\frac{\mathrm{m}}{\mathrm{s}}$$

3-2 Average Velocity and Average Speed

Average velocity is a vector quantity

The sign of the average velocity for the time interval t = 1.0 s to t = 3.0 s



The average velocity has both magnitude and direction.

→ Average velocity is a **vector quantity.**

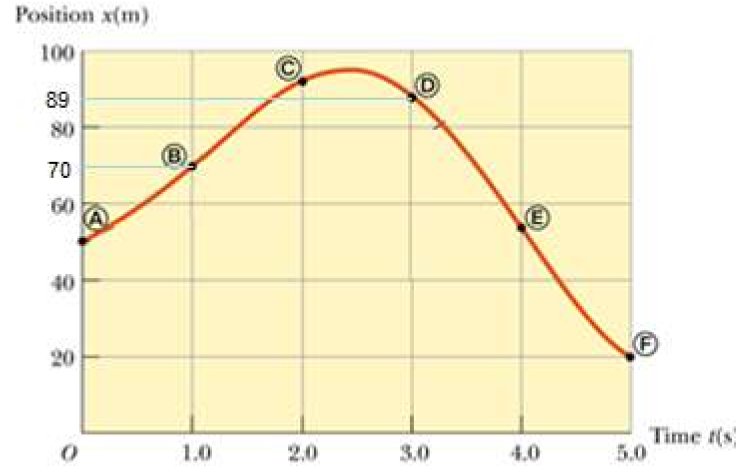
3-2 Average Velocity and Average Speed

Example

According to the graph

a) Calculate the average velocity of the motion between the points A and D.

b) Calculate average velocity between the points B and F.

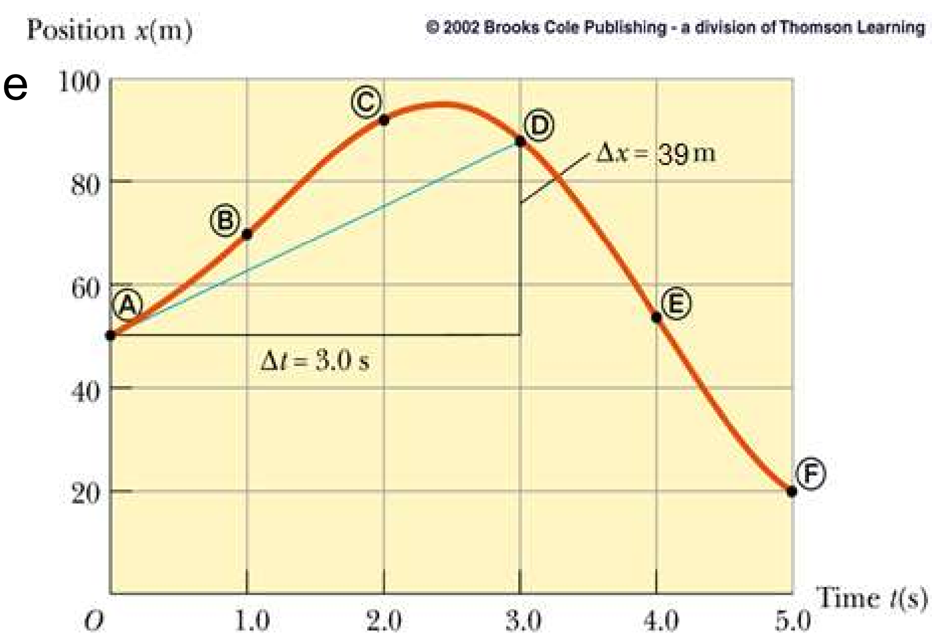


Solution

a) Average velocity equals the slope of the line joining the initial and final positions.

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{89-50m}{3-0s} = \frac{39m}{3s} = \underline{+13\,m/s}$$

b)



3-2 Average Velocity and Average Speed

Average speed

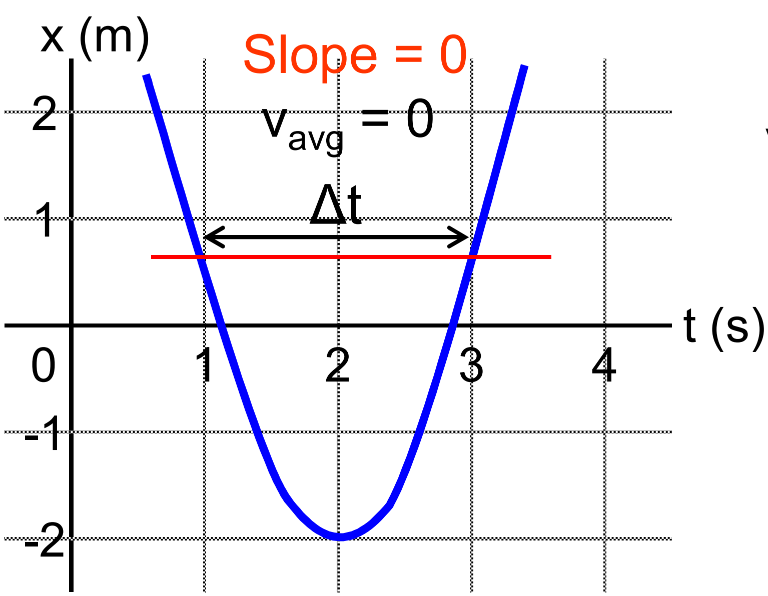
The average speed s_{avg} for a time interval is

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Time interval}}$$

$$s_{avg} = \frac{\text{Total distance}}{\Delta t}$$

Average speed is always positive. It does not include direction.

→ Average speed is a **scalar quantity**.



For the time interval t = 1.0 s to t = 3.0 s,

$$v_{avg} = 0.0\ \frac{m}{s}$$

$$s_{avg} = \frac{\text{Total distance}}{\Delta t} = \frac{5.0\ m}{2.0\ s} = 2.5\ \frac{m}{s}$$

Average speed is not always equal to the magnitude of average velocity.

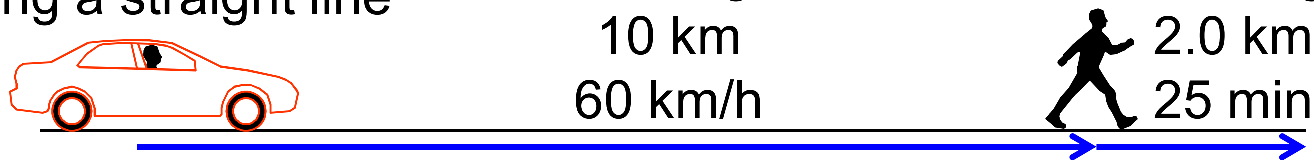
3-2 Average Velocity and Average Speed

Example

Trip along a straight line

Driving
10 km
60 km/h

Walking
2.0 km
25 min



What is your average velocity for the whole trip?

Solution

D = Driving
W = Walking

Displacement

$$\Delta x = \Delta x_D + \Delta x_W = 10 \text{ km} + 2.0 \text{ km} = 12 \text{ km}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{12 \text{ km}}{0.59 \text{ h}} = 20 \text{ km/h}$$

$$\Delta t = \Delta t_D + \Delta t_W = 0.17 \text{ h} + 0.42 \text{ h} = 0.59 \text{ h}$$

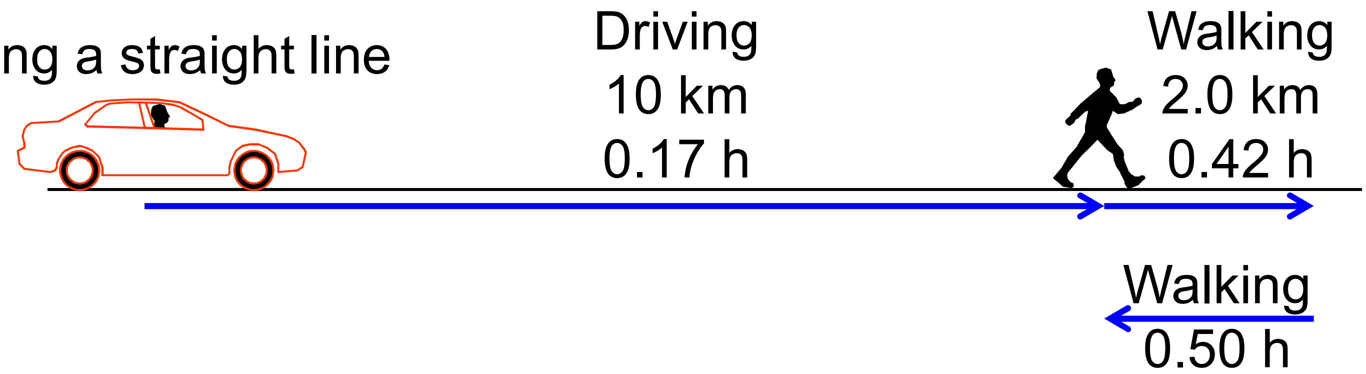
$$25 \text{ min} = 25 \text{ min}\left(\frac{1 \text{ h}}{60 \text{ min}}\right) = 0.42 \text{ h}$$

$$v_{avg,\,D} = \frac{\Delta x_D}{\Delta t_D} \rightarrow \Delta t_D = \frac{\Delta x_D}{v_{avg,\,D}} = \frac{10 \text{ km}}{60 \text{ km/h}} = 0.17 \text{ h}$$

3-2 Average Velocity and Average Speed

Example

Trip along a straight line



What is your average velocity and average speed for the whole trip?

Solution

In the negative direction

Displacement

$$\Delta x = 10\ \text{km} + 2.0\ \text{km} - 2.0\ \text{km} = 10\ \text{km}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10\ \text{km}}{1.09\ \text{h}} = 9.2\ \text{km/h}$$

$$\Delta t = 0.17\ \text{h} + 0.42\ \text{h} + 0.50\ \text{h} = 1.09\ \text{h}$$

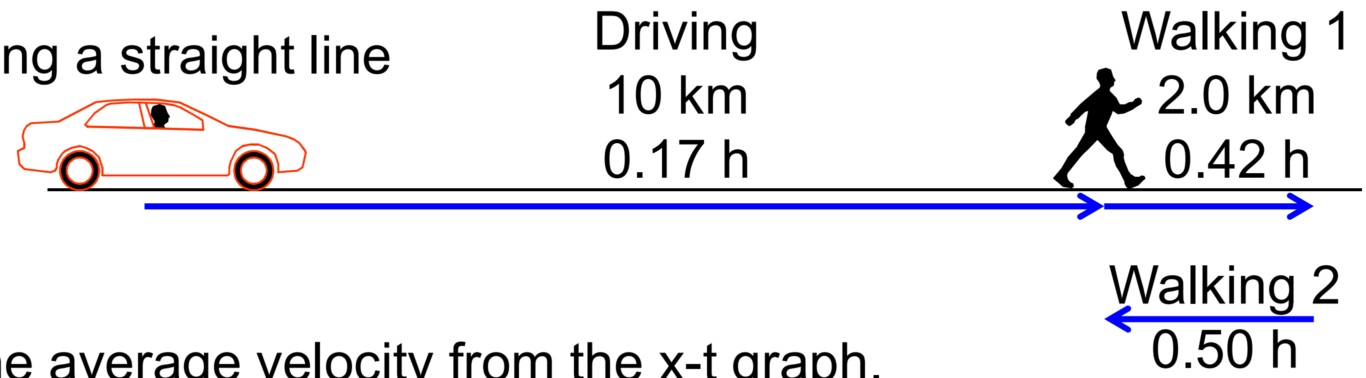
$$\text{Total distance} = 10\ \text{km} + 2.0\ \text{km} + 2.0\ \text{km} = 14\ \text{km}$$

$$S_{avg} = \frac{\text{Total distance}}{\Delta t} = \frac{14\ \text{km}}{1.09\ \text{h}} = 13\ \text{km/h}$$

3-2 Average Velocity and Average Speed

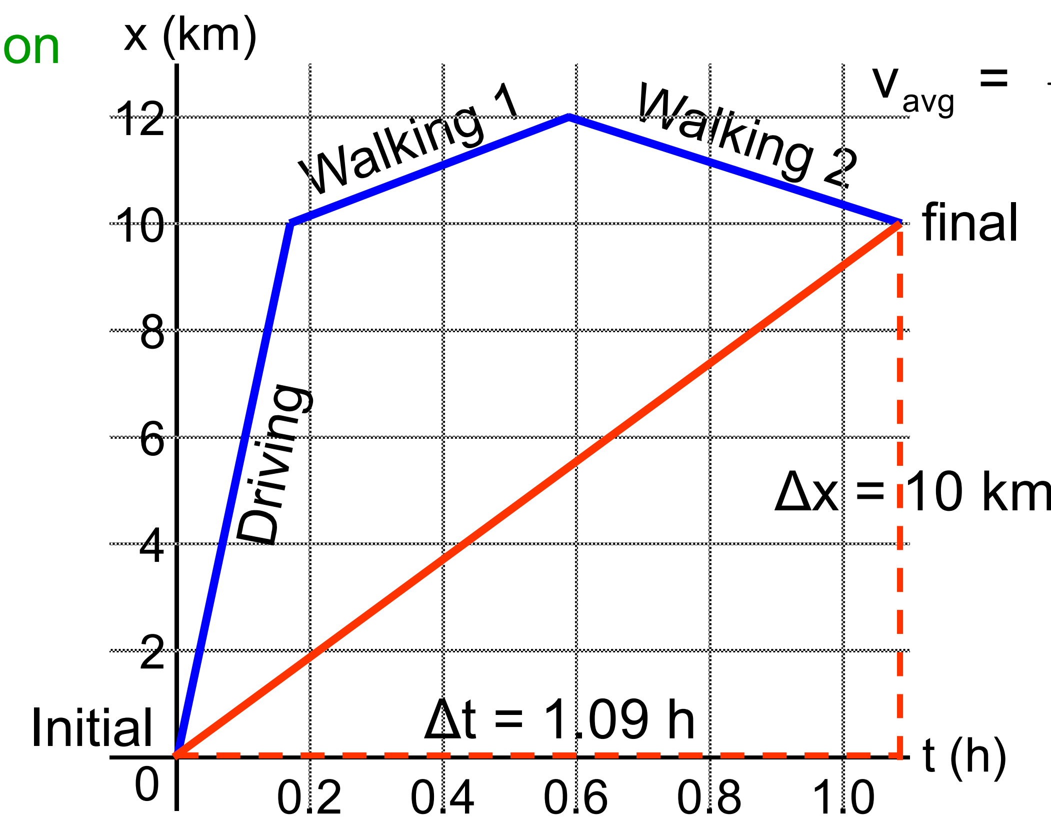
Example

Trip along a straight line



Find the average velocity from the x-t graph.

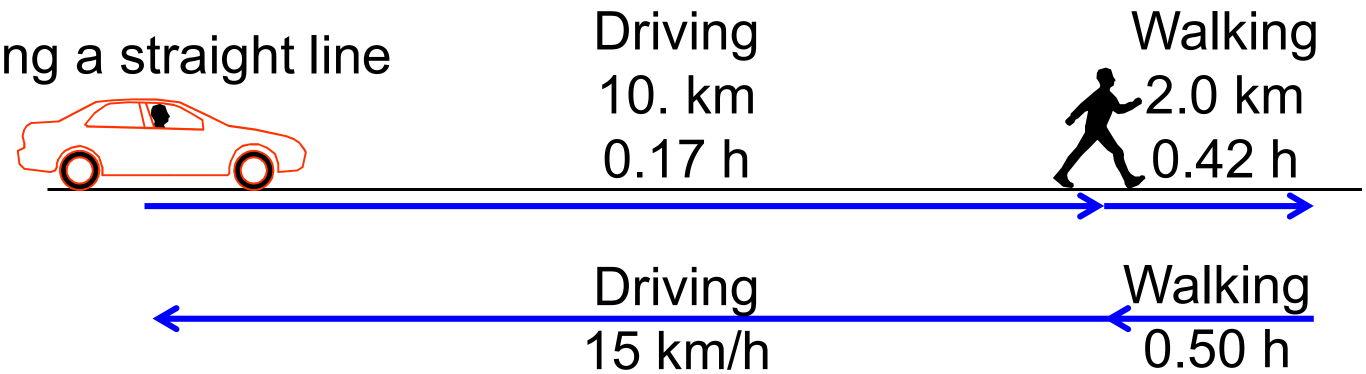
Solution



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10\ \text{km}}{1.09\ \text{h}} = 9.2\ \text{km/h}$$

3-2 Average Velocity and Average Speed
Checkpoint

Trip along a straight line



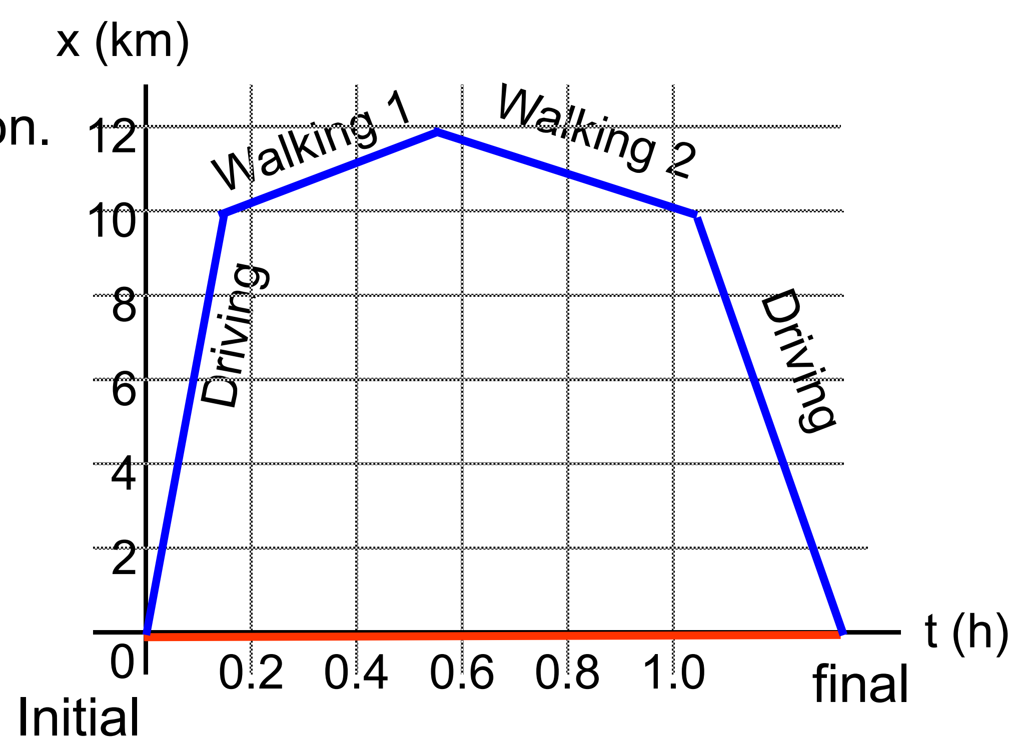
What is your average velocity?

Solution

Let`s draw the graph of the motion.

$$v_{avg} = \frac{\Delta x}{\Delta t} = 0 \text{ m/s}$$

Since the displacement is zero.

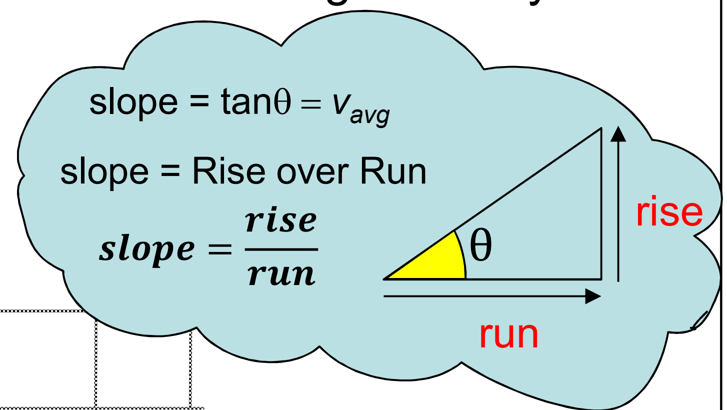


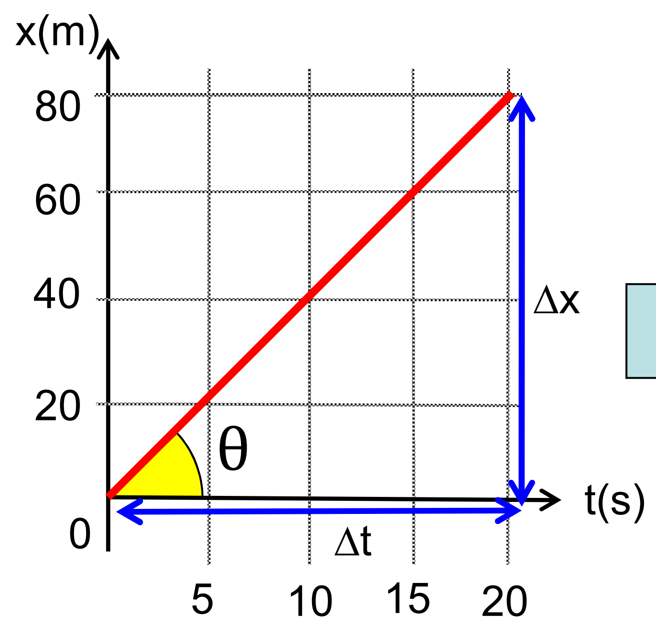
3-2 Average Velocity and Average Speed
Drawing velocity – time graph from position time graph

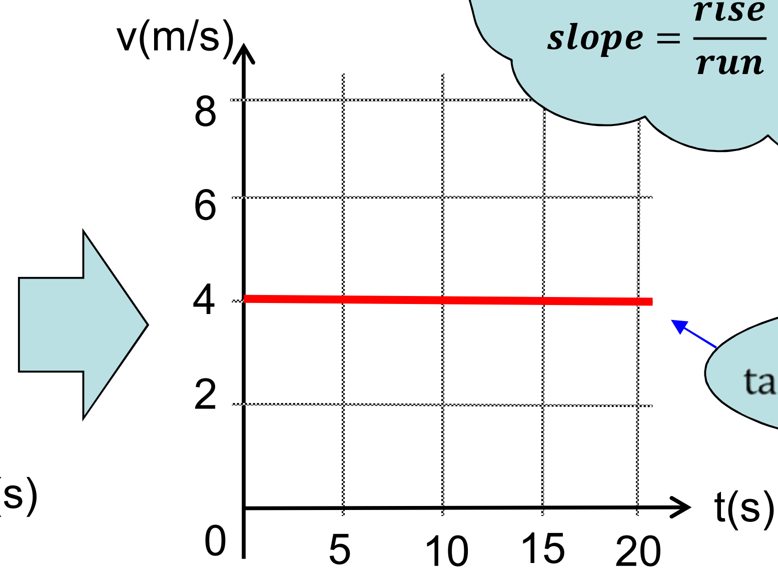
An object moves 80 meters in 20 seconds as in the graph. Draw the velocity – time graph of the motion.

The slope of a line in x-t graph gives the avarage velocity of the motion.
The slope is calculated by tangent equation.
Here, in this graph, tanθ gives the slope which is equal to the avarage velocity.

$$\tan\theta = \frac{\Delta x}{\Delta t} = v_{avg} = \frac{\Delta x}{\Delta t} = \frac{80-0}{20-0} = 4\ m/s$$





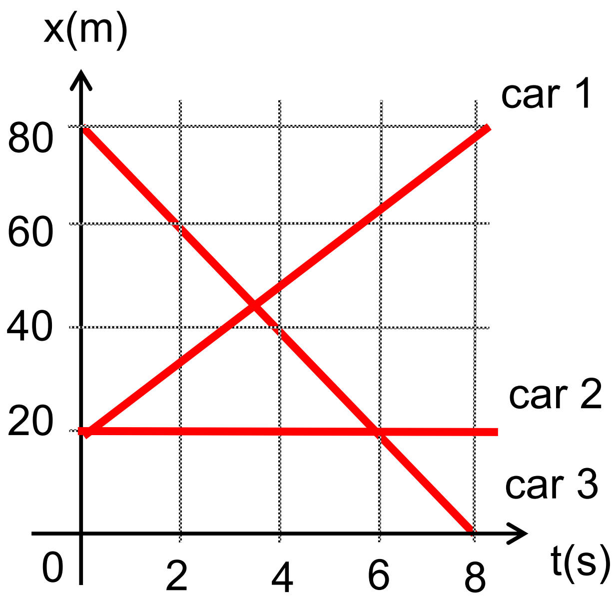


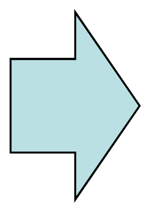
$$\tan\theta = v_{avg} = 4m/s$$

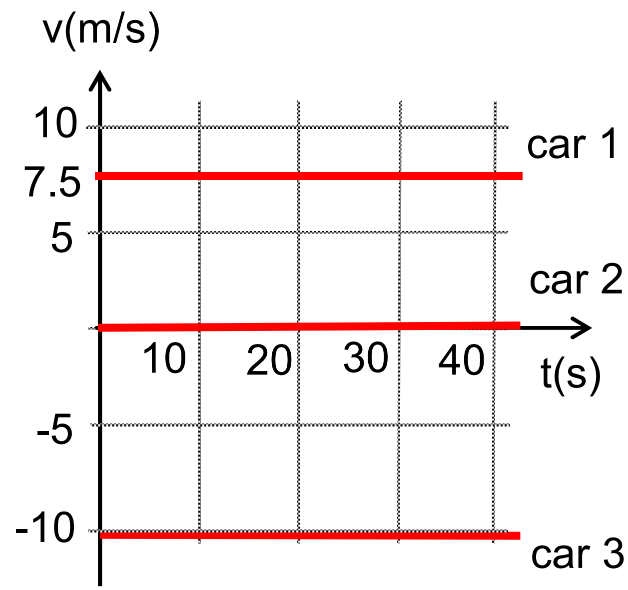
3-2 Average Velocity and Average Speed

Drawing velocity – time graphs from position time graphs

If the velocity is constant, the position – time graph of this motion is a straight line. The slope of the line indicates the velocity.





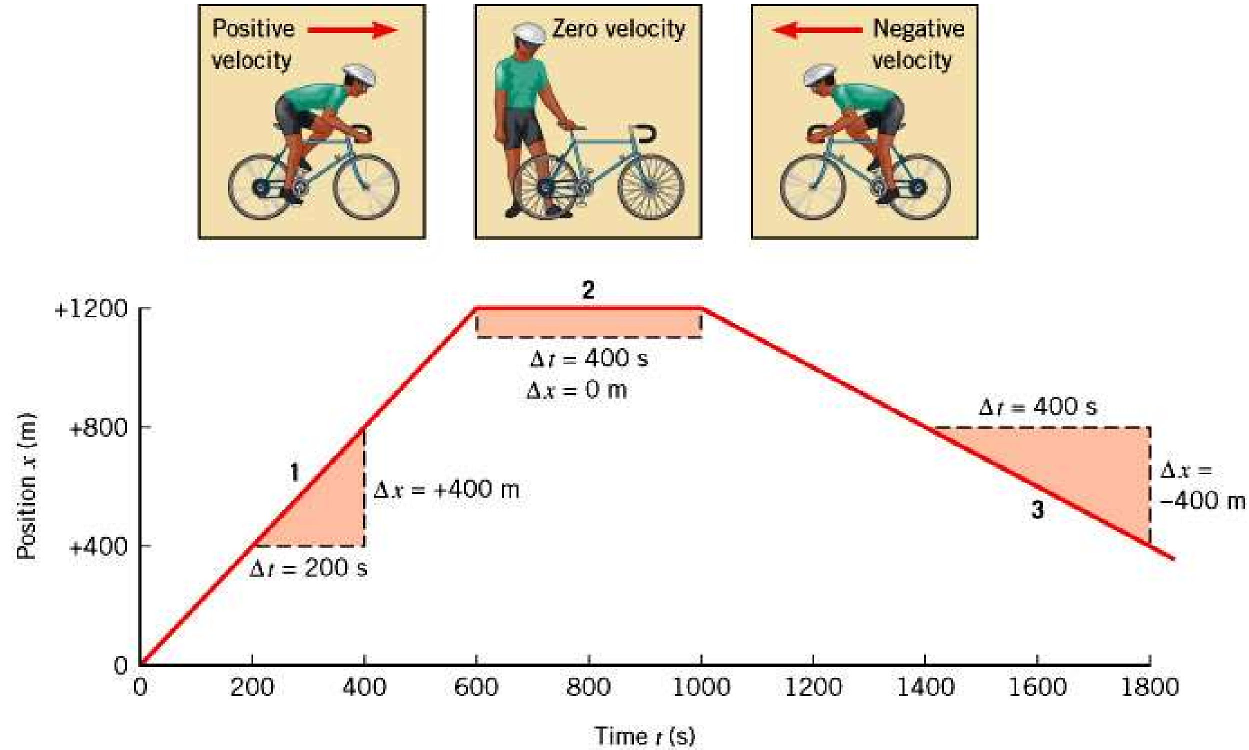


- ✓ **Car 1: positive slope = positive velocity**
- ✓ **Car 2: zero slope= zero velocity = object is at rest**
- ✓ **Car 3: negative slope = negative velocity**

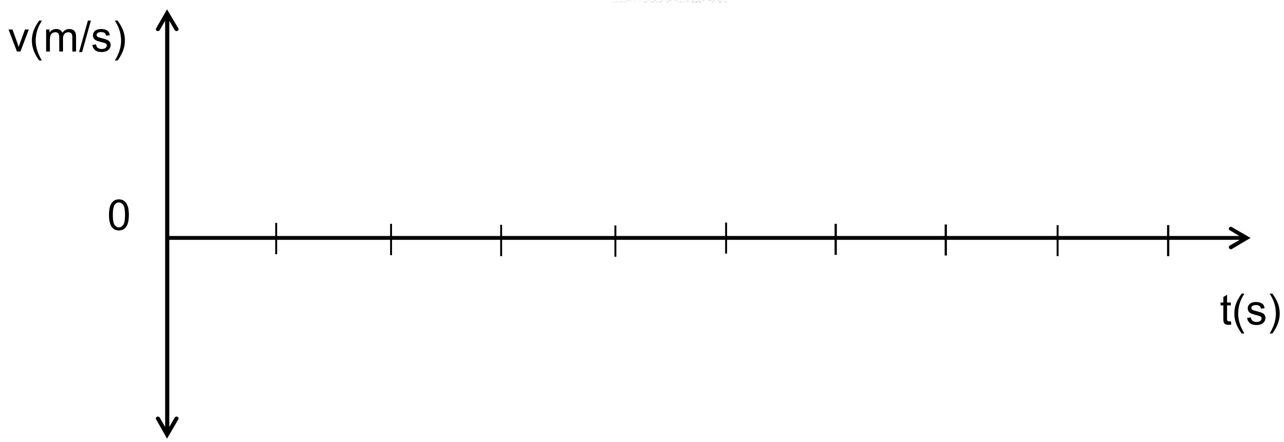
3-2 Average Velocity and Average Speed

Example: Drawing velocity –time graph

The graph shows the motion of a bicycle rider with time. Draw the velocity versus time graph for this motion.



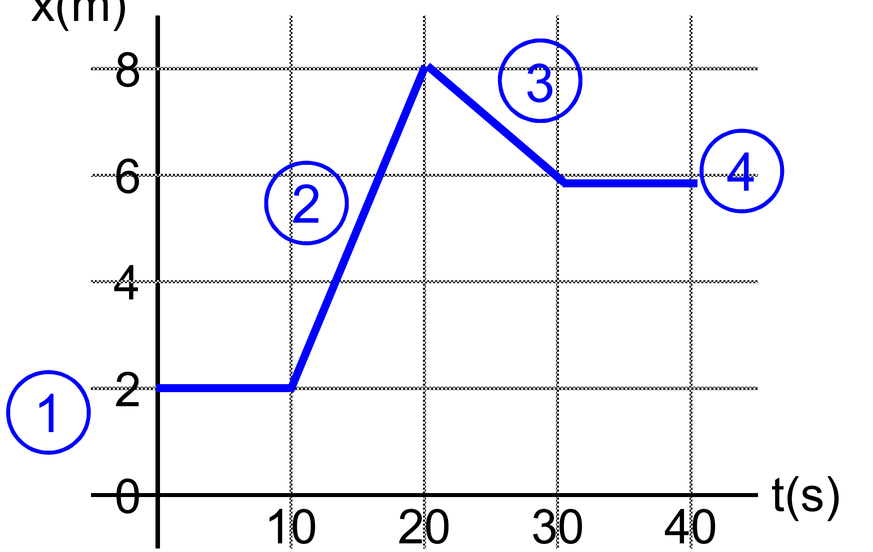
Solution:

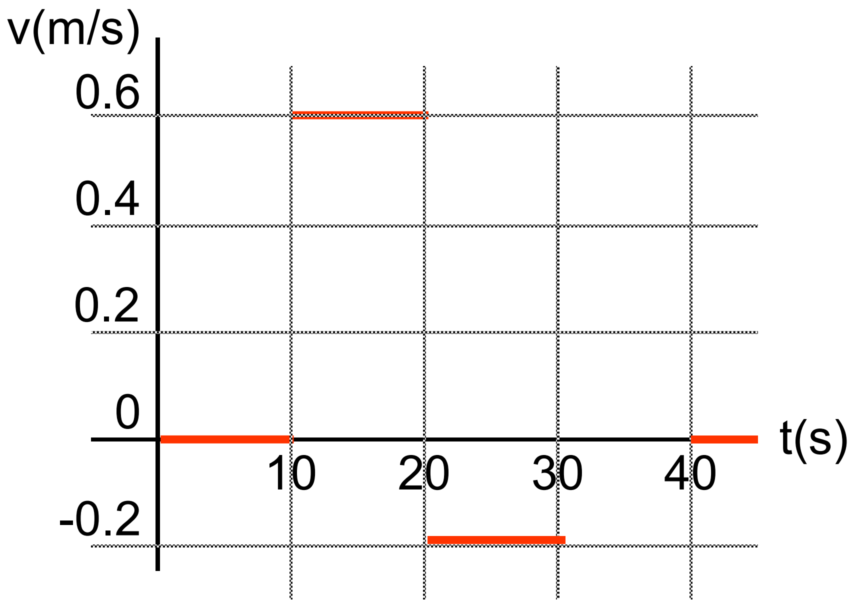


3-2 Average Velocity and Average Speed

Example: Drawing velocity –time graph

Draw velocity versus time graph.





Solution

$$v = \frac{dx}{dt} = \text{slope}$$

For 1 and 4,

slope = 0

For 2,

$$\text{slope} = \frac{8.0 - 2.0}{20 - 10}\,\frac{m}{s}$$

$$= \frac{6.0}{10}\,\frac{m}{s} = 0.6\,\frac{m}{s}$$

For 3,

$$\text{slope} = \frac{6.0 - 8.0}{30 - 20}\,\frac{m}{s}$$

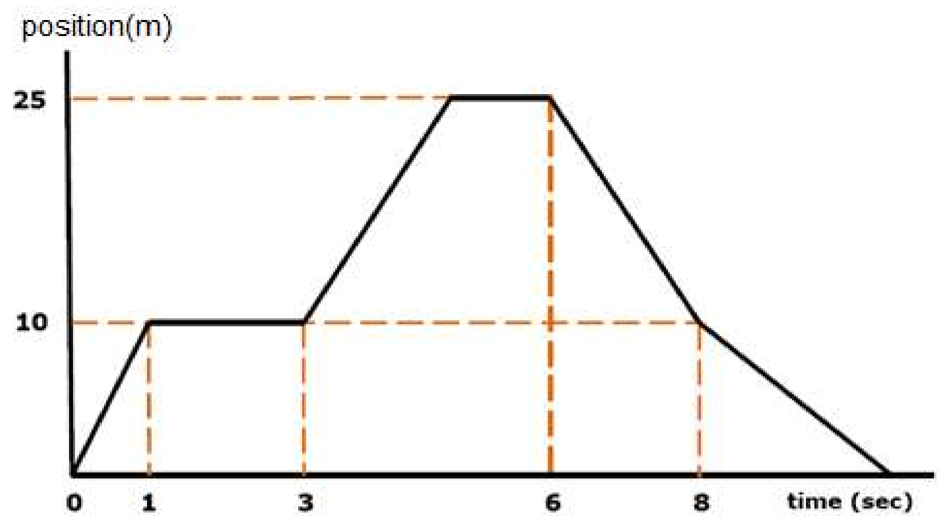
$$= \frac{-2.0}{10}\,\frac{m}{s} = -0.2\,\frac{m}{s}$$

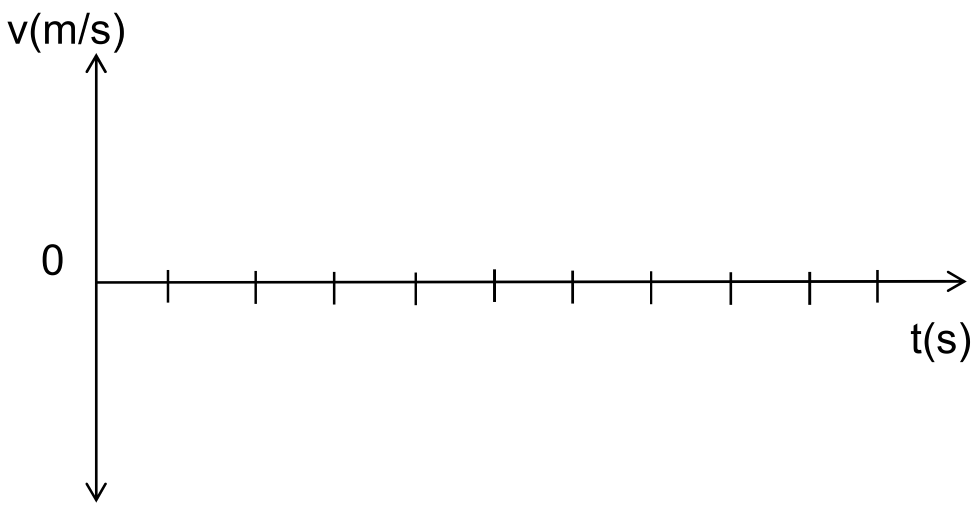
3-2 Average Velocity and Average Speed

Example: Drawing velocity –time graph

Sketch the velocity-time graph for the following distance-time graph.

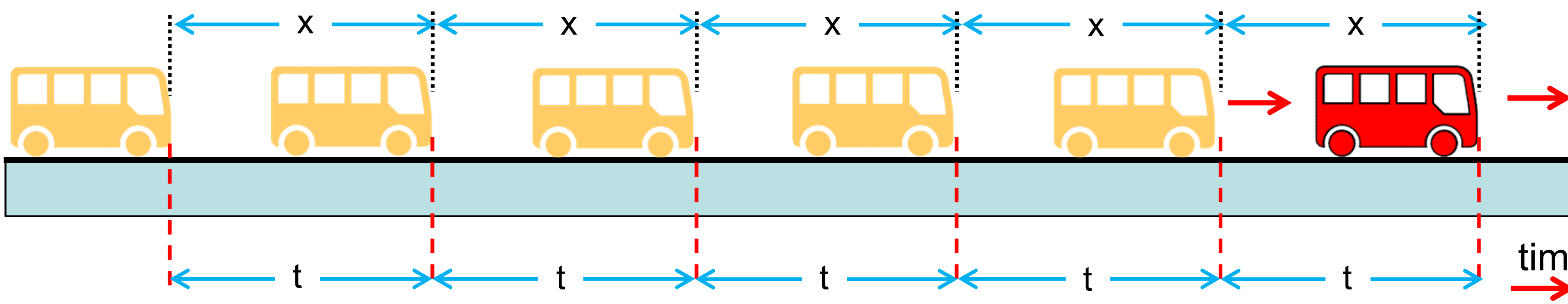
Solution:





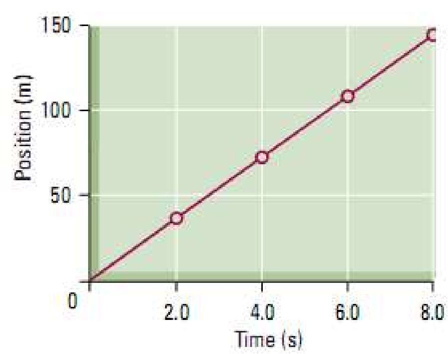
3-2 Average Velocity and Average Speed
Uniform motion

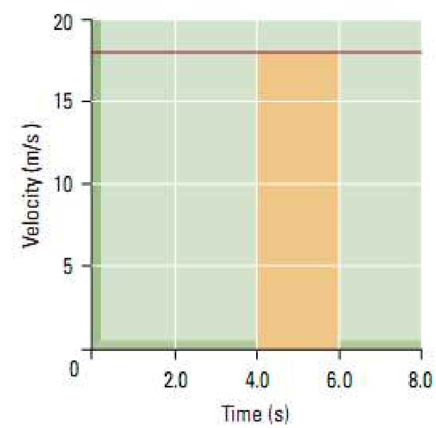
If an object covers equal distances in equal time intervals then this motion is called in **uniform motion**. A car, a bus, a plane, a train, a bicyle with steady(constant) speed are some examples of uniform motion.



Example: The table below shows the change of an object`s position with respect to time. According to this table, draw the position-time graph, and velocity-time graph of the motion. What is the velocity of the object? Is it a uniform or non-uniform motion?

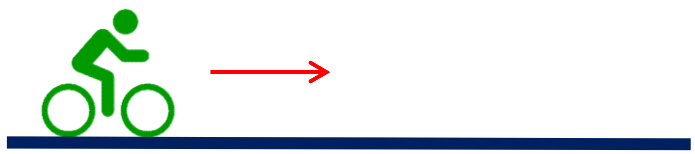
Time (s)	Position (m [W])
0.0	0
2.0	36
4.0	72
6.0	108
8.0	144

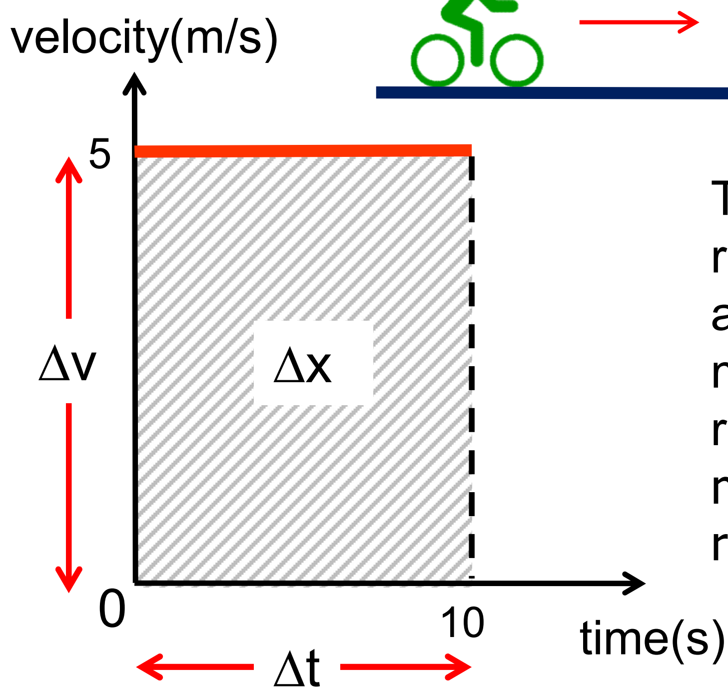




3-2 Average Velocity and Average Speed
Calculating area under a graph

The area the under velocity-time graph gives the displacement of the motion.





The graph shows the velocity of a bicyle rider with respect to time. If we want to calculate the shaded area under the graph in the figure, we should multiply the sides of the rectangle. The sides of the rectangle are Δv, and Δt. The result of this multiplicaton gives us the displacement of the rider during the time interval.

$$\boxed{\Delta x = \Delta v \times \Delta t}$$

Example: Calculate the displacement of the rider given in the graph.

$\Delta v = 5$ m/s

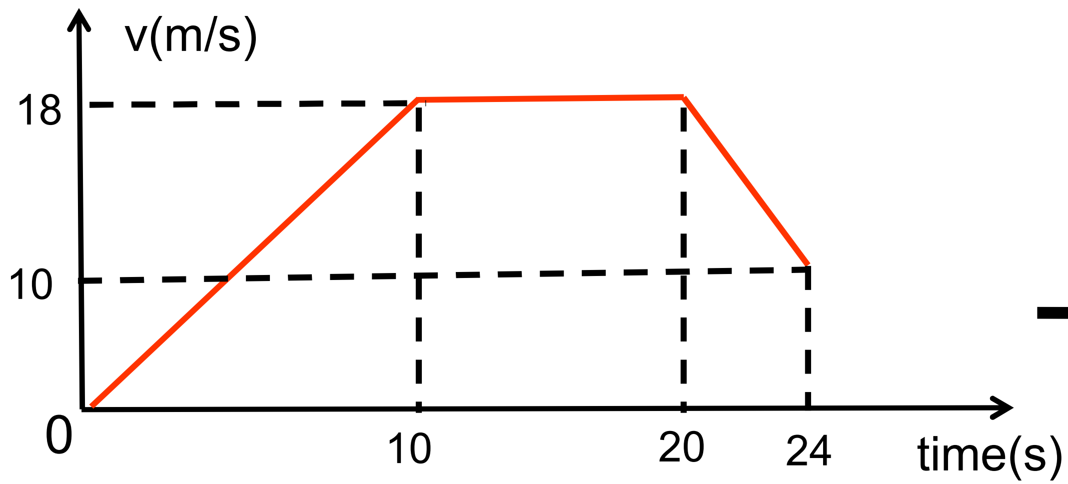
$\Delta t = 10$ s

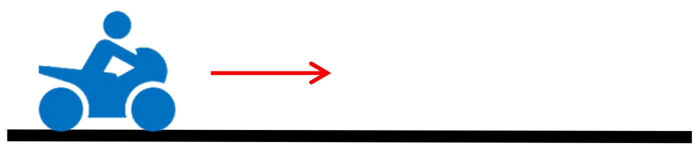
$\Delta x = \Delta v \times \Delta t$

$x = 5 \times 10 = 50$ m

3-2 Average Velocity and Average Speed
Calculating area under a graph

The graph shows the change in velocity of a motorbike with time. Plot the position time graph of this motion.





3-2 Average Velocity and Average Speed
Example

Use the graphs to answer questions.

1. Which graph represents an object moving with a constant positive velocity?

   A. I   B. II   C. III   D. IV

2. Which graph represents an object at rest?

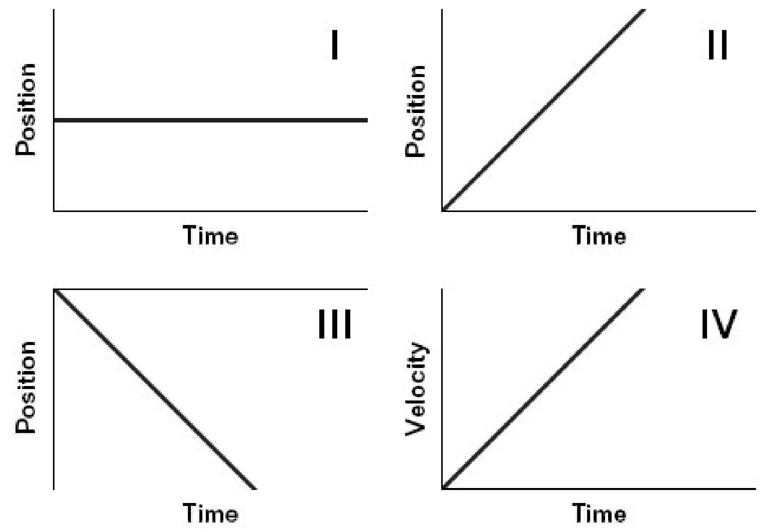
   A. I   B. II   C. III   D. IV

3. Which graph represents an object moving with a constant negative velocity?

   A. I   B. II   C. III   D. IV

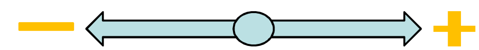
4. Which graph represents an object moving with an increasing velocity?

   A. I   B. II   C. III   D. IV



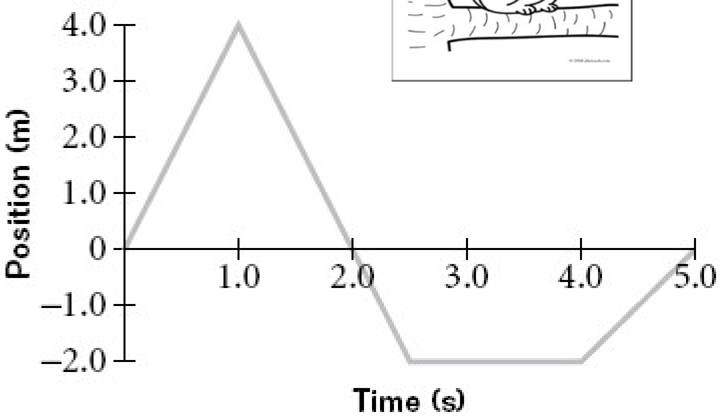
3-2 Average Velocity and Average Speed
Example

Use the position-time graph of a squirrel running along a clothesline to answer questions.



1. What is the squirrel's velocity between 1.0 and 2.0 s?

A. –6.0 m/s B. –4.0 m/s
C. +4.0 m/s D. +2.0 m/s



2. What is the squirrel's average velocity during the time interval between 0.0 s and 3.0 s?

A. –2.0 m/s B. –0.67 m/s
C. 0.0 m/s D. +0.53 m/s

3. What is the squirrel's speed between 0.0 and 5.0 s?

A. 0.0 m/s B. 2.4 m/s C. 4.0 m/s D. 12.0 m/s

4. What is the squirrel's average velocity between 0.0 and 5.0 s?

A. 0.0 m/s B. 2.4 m/s C. 4.0 m/s D. 12.0 m/s

3-3 Instantaneous Velocity and Speed
Velocity

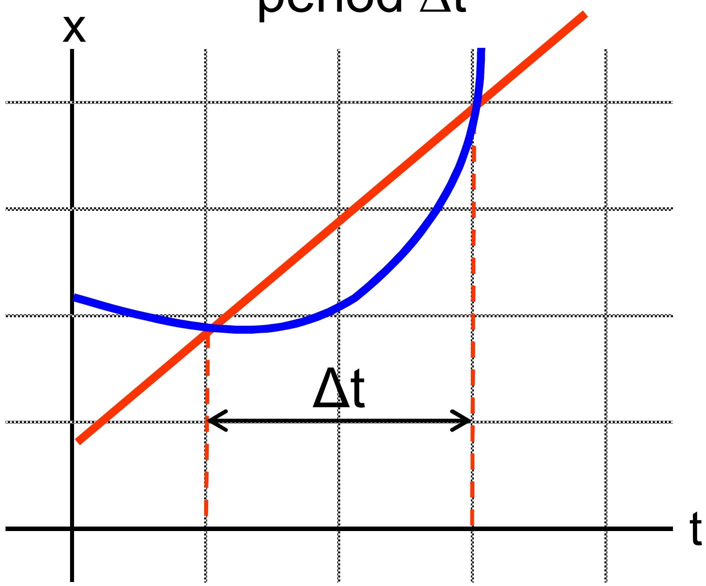
The instantaneous velocity v at a given instant is:

$$\text{velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

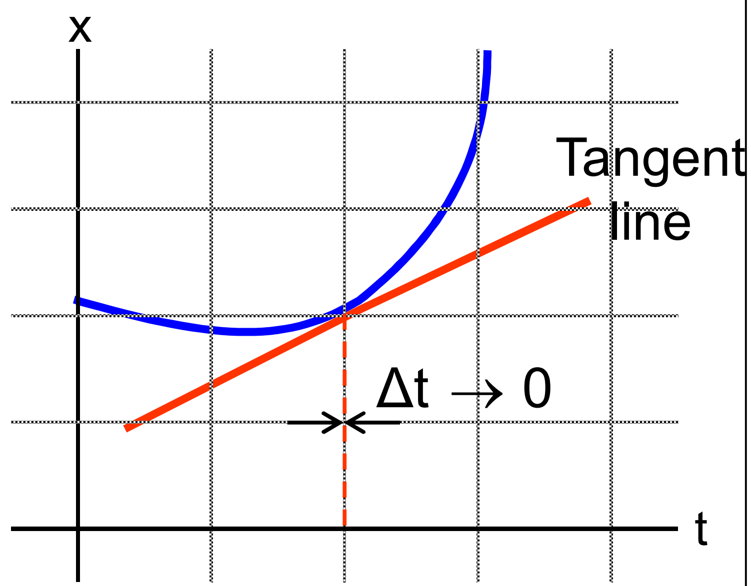
$$v = \frac{dx}{dt}$$

velocity ≡ instantaneous velocity

Average velocity over a time period Δt



Velocity at an instant



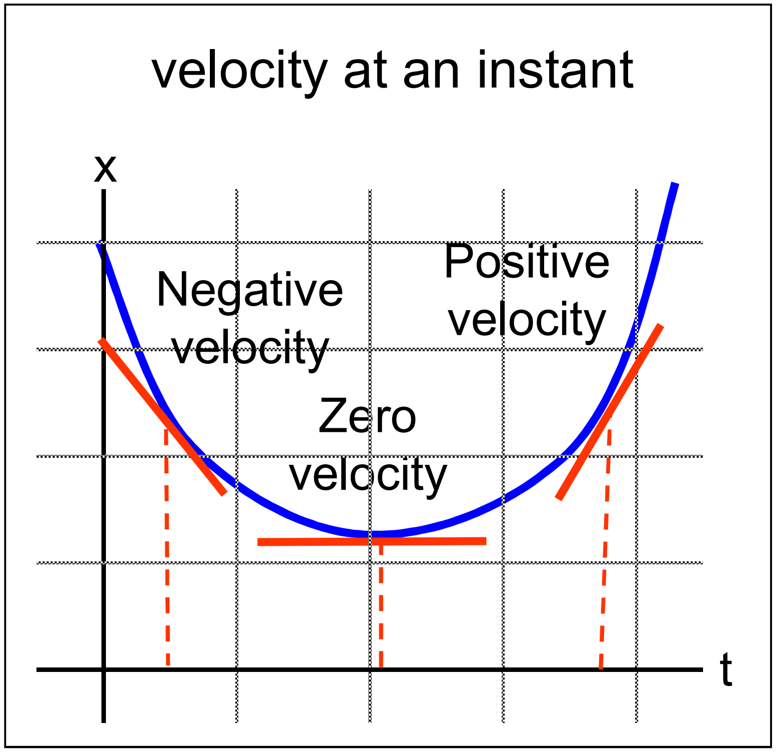
3-3 Instantaneous Velocity and Speed
Velocity is the slope of x-t curve

The velocity v at a given instant is:

$$v = \frac{dx}{dt}$$

Velocity is the derivative of x with respect to t.

Velocity is a vector quantity. But speed is a scalar quantity. Therefore speed doesn't have direction, it is always positive. The speed at a given instant is the magnitude of the velocity.



$$\text{speed} = |v|$$

$v = -4$ m/s → speed = 4 m/s

$v = 4$ m/s → speed = 4 m/s

3-3 Instantaneous Velocity and Speed

Example

The position of a particle moving on an x axis is given by

$$x = 8.3 + 5.0\,t - 3.0\,t^3,$$

with x in meters and t in seconds.

Find the velocity at t = 1.0 s.

Solution

$$v = \frac{dx}{dt} = \frac{d}{dt}(8.3 + 5.0\,t - 3.0\,t^3) = 5.0 - (3)(3.0)\,t^2$$

$$= 5.0 - 9.0\,t^2$$

At t = 1.0 s,

$$v = 5.0 - 9.0\,(1.0)^2 = -4.0 \text{ m/s.}$$

At t = 1.0 s, the particle is moving in the negative direction with a speed of 4.0 m/s.

3-3 Instantaneous Velocity and Speed

Example

The position of a particle moving on an x axis is given by **$x = 3t^2 - 5t + 20$**

with x in meters and t in seconds.

A. What is the position of the particle at t = 3 seconds?

B. What is the displacement of the particle during the time interval t = 2 and t = 4 s?

C. Find the velocity at t = 4.0 s.

Solution

A) At t = 3.0 s, x = ………..

B) Displacement = final position – initial position

………….

C) $v = \dfrac{dx}{dt}$ ………….

At t = 4.0 s, ……

At t = 4.0 s, the particle is moving in the …………… direction with a speed of …………….. .

3-3 Instantaneous Velocity and Speed

Checkpoint

The following equations give the position x of a particle in four situations. x in meters, t in seconds, and $t > 0$.

$x = 2t - 3$
$x = -3t^2 - 1$
$x = 2/t^2$
$x = -2$

In which situation is the velocity of the particle constant?
In which situation is v in the negative x direction?

Solution

$v = 2$	constant	
$v = -6t$	variable	neagtive
$v = -4/t^3$	variable	negative
$v = 0$	constant	