Chapter 2 – Part A
Vectors

2-1 Coordinate Systems
2-2 Physical Quantities
2-3 Adding Vectors Geometrically
2-4 Components of Vectors
2-5 Adding Vectors by Components

2-1 Coordinate Systems

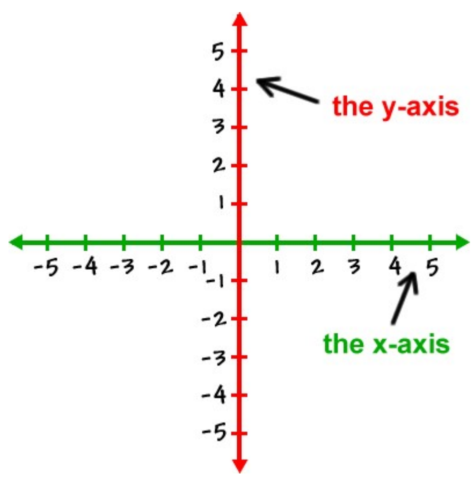
Coordinate system is used to describe the position of a point in space.

Coordinate system (frame) consists of

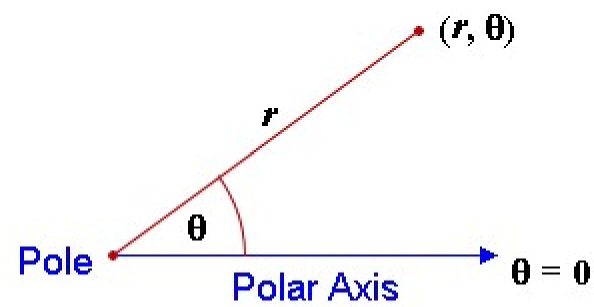
- a fixed reference point called the origin
- specific axes with scales and labels
- instructions on how to label a point relative to the origin and the axes

Types of Coordinate Systems

- Cartesian Coordinate System



- Polar Coordinate System



2-1 Coordinate Systems

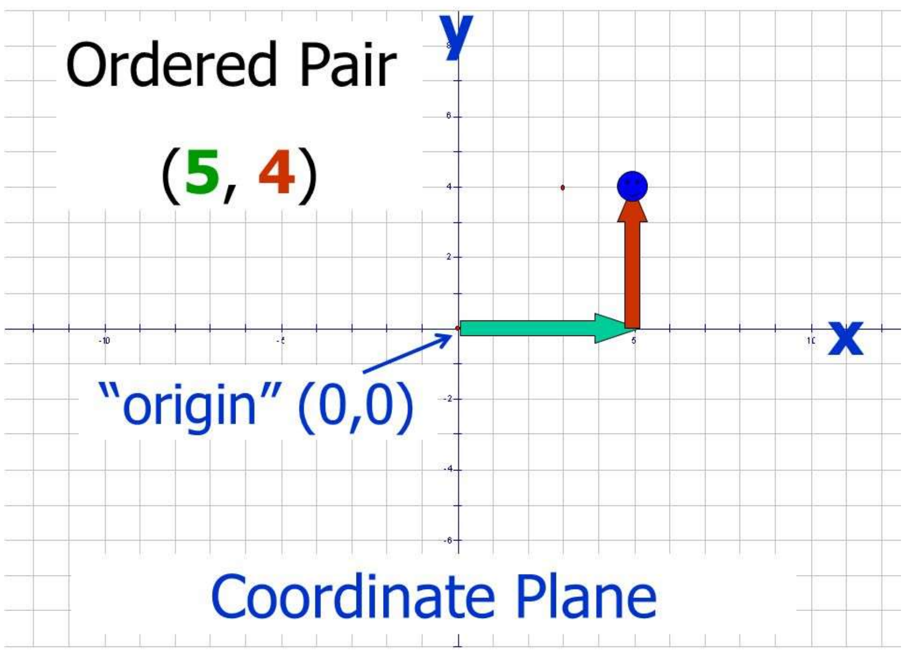
Cartesian Coordinate System

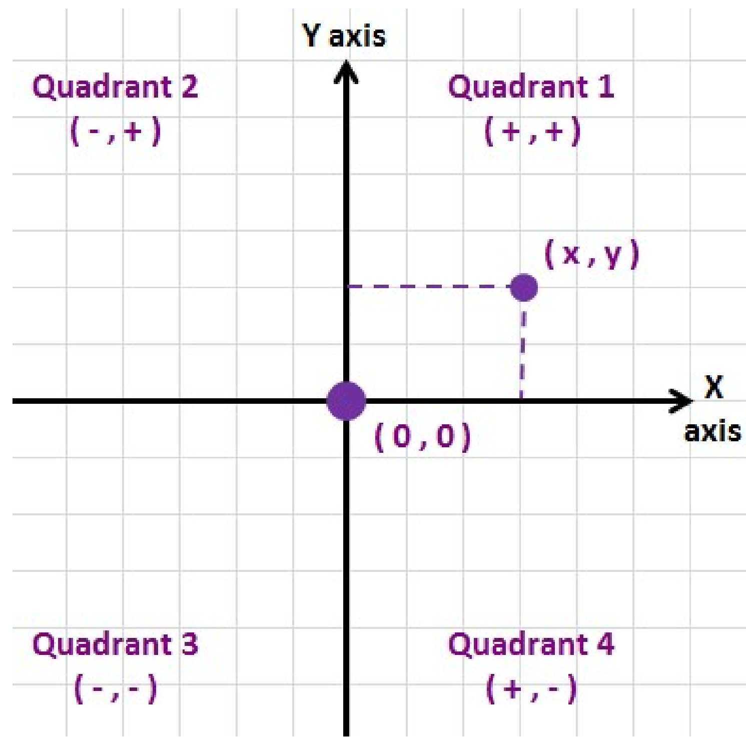
- It is also called rectangular coordinate system
- x- and y- axes, points are labeled (x, y)

Cartesian coordinates can go:

- left-right, and
- up-down

so any position needs two numbers



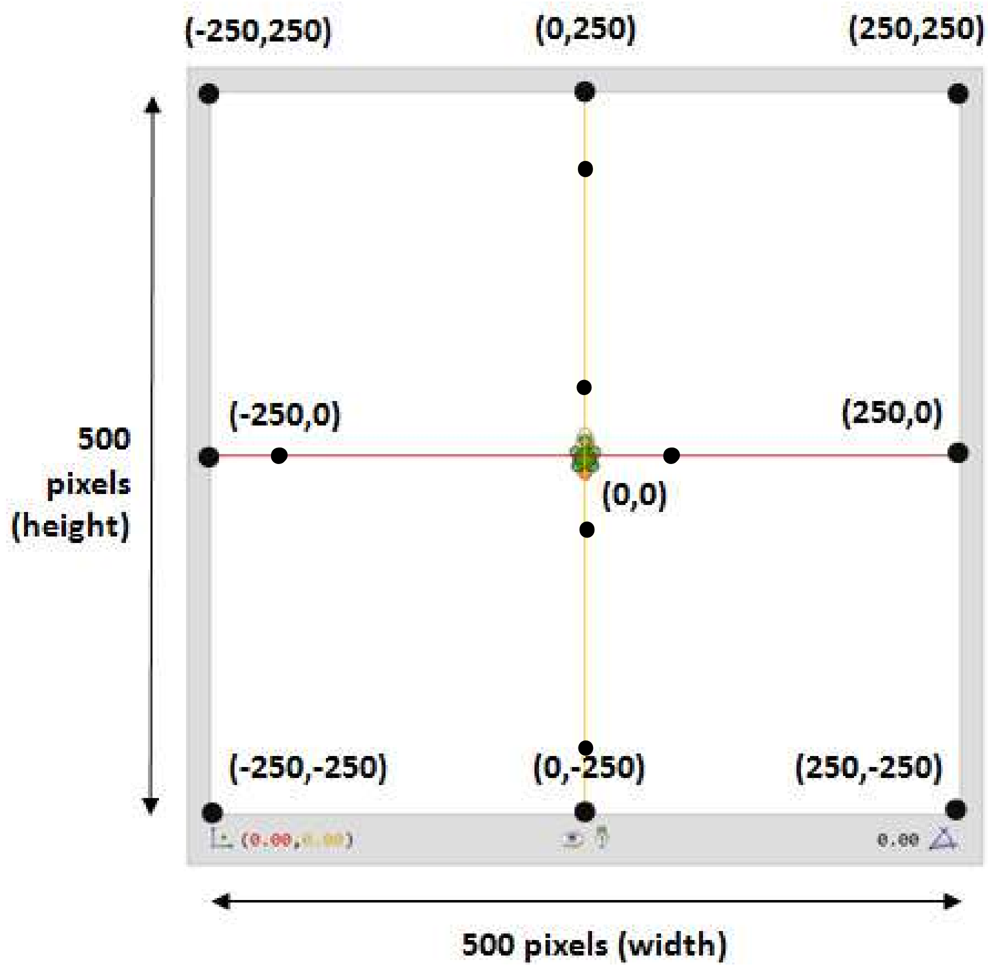


2-1 Coordinate Systems

Example 1

The turtle on the computer screen (500 pixels x 500 pixels) starts from the origin and first moves to the points (200, 0) then to (200, 150), then to (-200, 150) then to (-200,-100) then to (0, -100) and stops.

A) Draw the path of the turtle B) Find the distance covered by the turtle.



2-1 Coordinate Systems

Cartesian Coordinate System - **3 Dimensions**

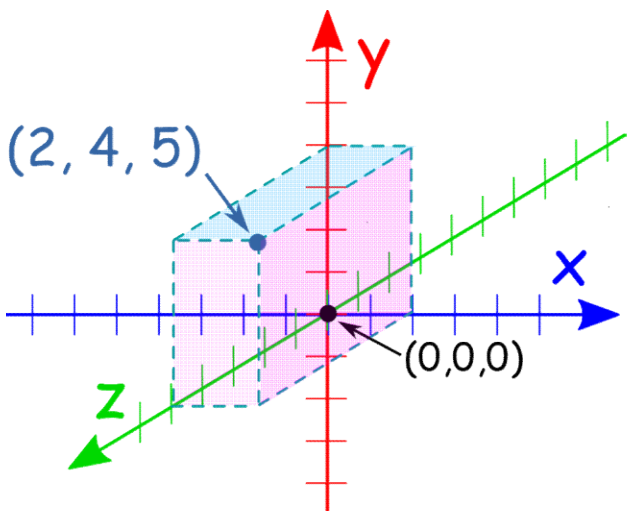
How do we locate a spot in the real world (such a a lamp hanged from the ceiling)?

We need to know:

- left-right,
- up-down, and
- forward-backward,

that is three numbers, or 3 dimensions!

- **Cartesian coordinates can be used for locating points in 3 dimensions.**



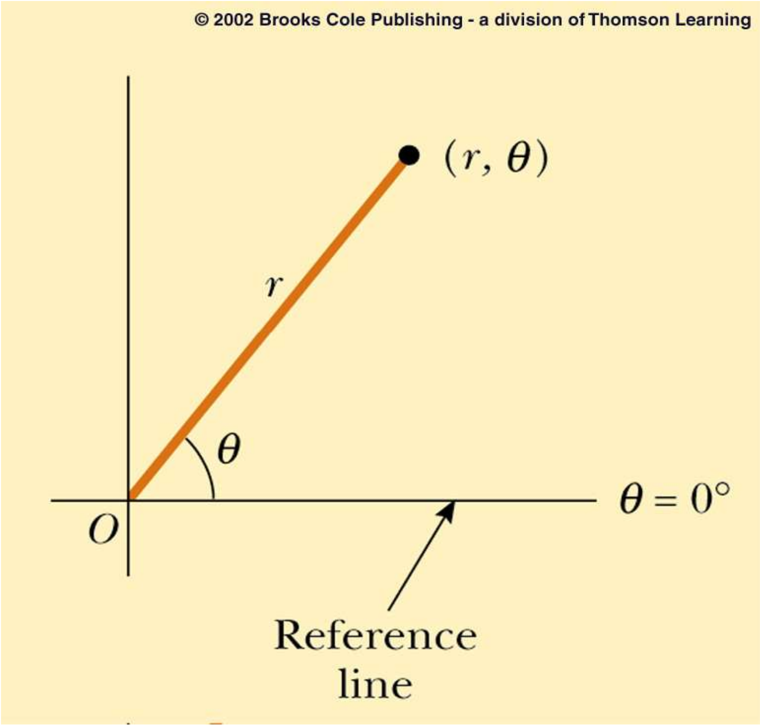
In this figure the point (2, 4, 5) is shown in three-dimensional Cartesian coordinates.

2-1 Coordinate Systems

Polar Coordinate System

- origin and reference line are noted
- point is distance r from the origin in the direction of angle θ, ccw from reference line
- points are labeled (r,θ)

➢ **When we use Polar Coordinates we mark a point by how far away from the origin, and what angle it is.**



2-1 Coordinate Systems

Math Review: Trigonometry

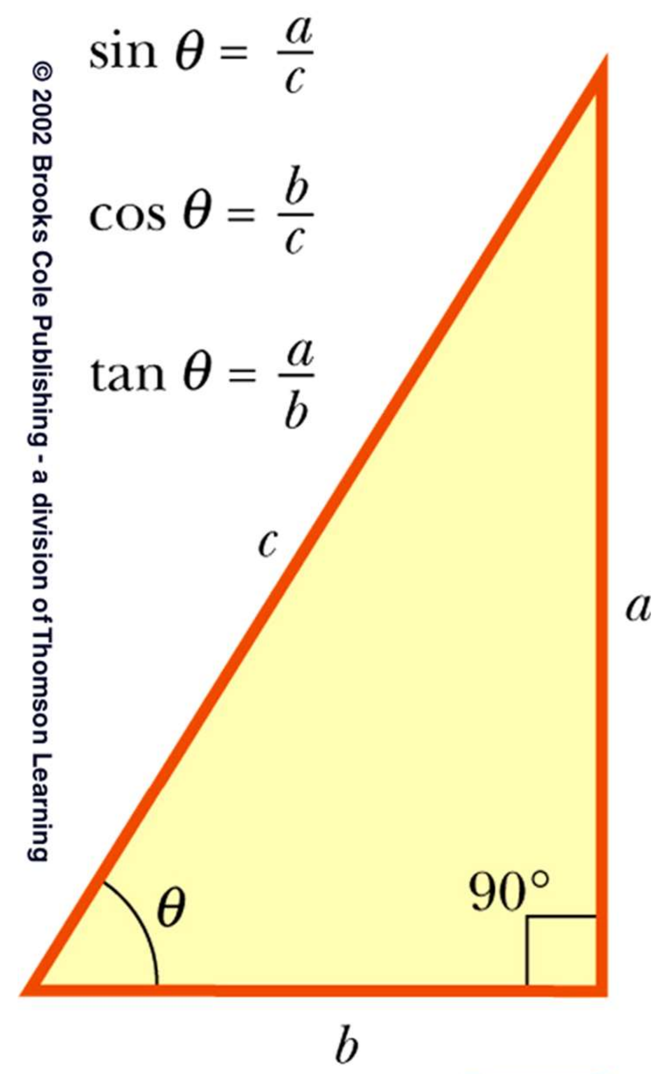
$$\sin\theta = \frac{opposite\ side}{hypotenuse}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse}$$

$$\tan\theta = \frac{opposite\ side}{adjacent\ side}$$

- **Pythagorean Theorem**

$$a^2 + b^2 = c^2$$



2-1 Coordinate Systems

To Convert from Polar to Cartesian

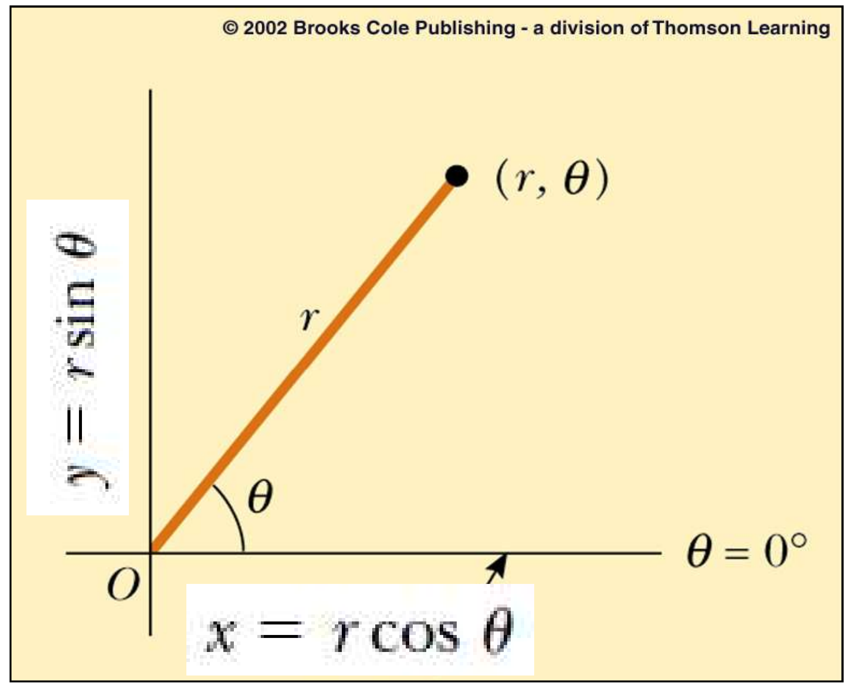
If we know a point in Polar Coordinates (r, θ), and we want it in Cartesian Coordinates (x,y) we solve a right triangle with a known side and angle:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

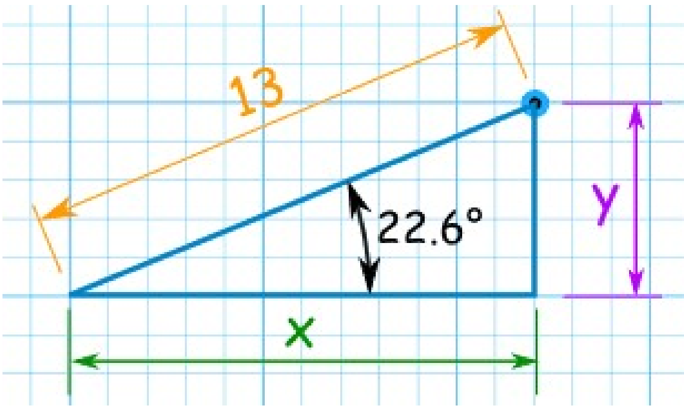


2-1 Coordinate Systems

Example 2 - To Convert from Polar to Cartesian

What is (13, 22,6°) in Cartesian Coordinates?

x = ? ; y = ?

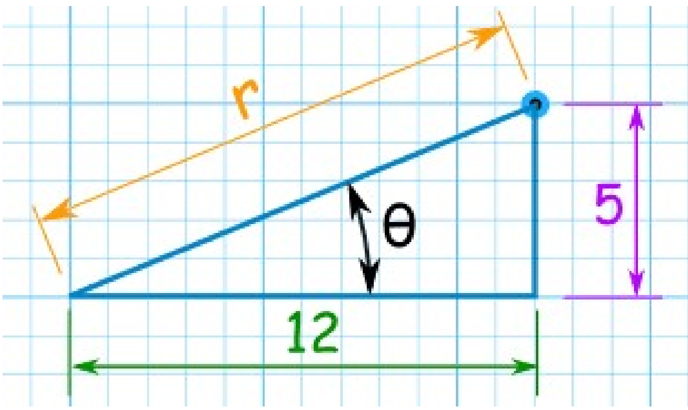


2-1 Coordinate Systems

Example 3 - To Convert from Cartesian to Polar

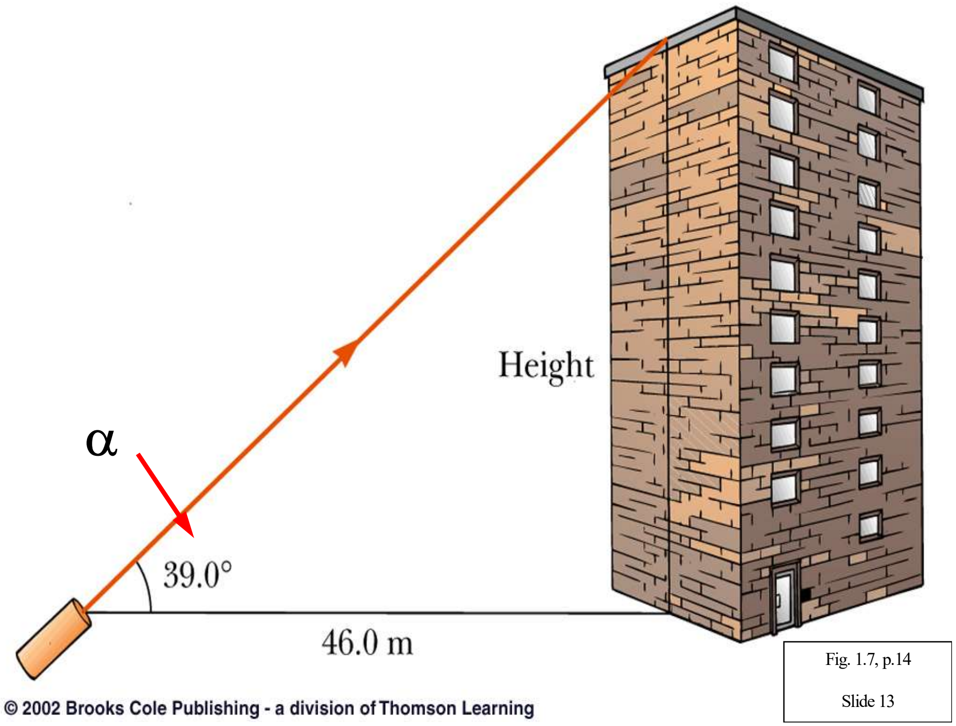
What is (12,5) in Polar Coordinates?

r = ? Θ = ?



2-1 Coordinate Systems
Example 4

Using the givens in the figure find the hight of the building.



Known: angle and one side
Find: another side

Key: tangent is defined via two sides!

Solution:

$$\tan\alpha = \frac{height\ of\ building}{dist.},$$

$$height = dist. \times \tan\alpha = (\tan 39.0^{\circ})(46.0\ m) = 37.3\ m$$

2-1 Coordinate Systems

Example 5

1) If the rectangular coordinates of a point are given by (3, y) and its polar coordinates are (r, 60^o), determine y and r.

(sin 30 = cos 60 = ½ ; sin 60 = cos 30 =$\sqrt{3}/2$)

Answer:
r = 6 m ; y = 3 $\sqrt{3}$ m

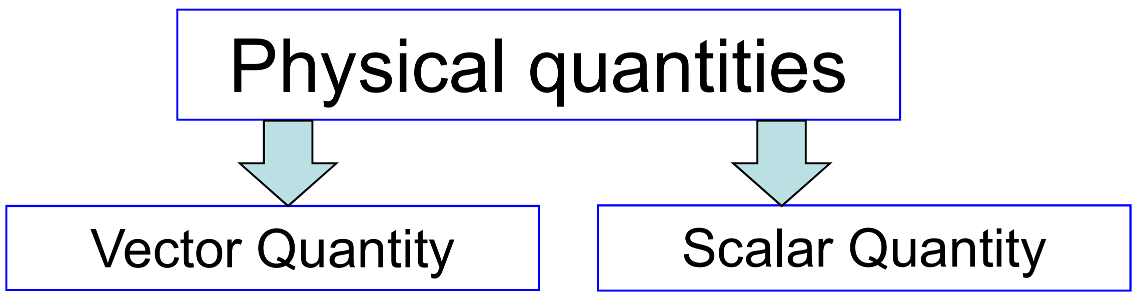
2) Calculate x and θ, if the polar coordinates of point P are given by (5 m,θ) and its rectangular coordinates are (x,3 m).

Answer:
x = 4 m ; θ = 37^o

2-2 Physical quantities
Introduction

A physical quantity is a property of a material or system that can be quantified by measurement.

Physical quantities can be grouped into two categories: Vector and Scalar quantities



2-2 Physical quantities
Scalar Quantities

- Scalar quantities are completely described by magnitude (bigness, size, amount,) only. (temperature, length,…). A scalar quantity has no direction.

Examples: temperature, length, mass, size

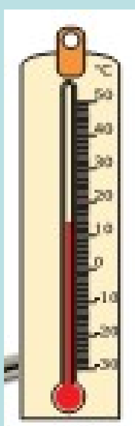
- A scalar quantity is specified by a value with appropriate unit.

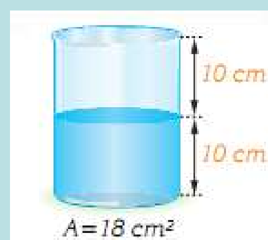
Example: Temperature is 25^0 C.

- Scalars can be negative.

Temperature = - 2^0 C means that it is 2 degrees below zero. This negative sign has nothing to do with direction.

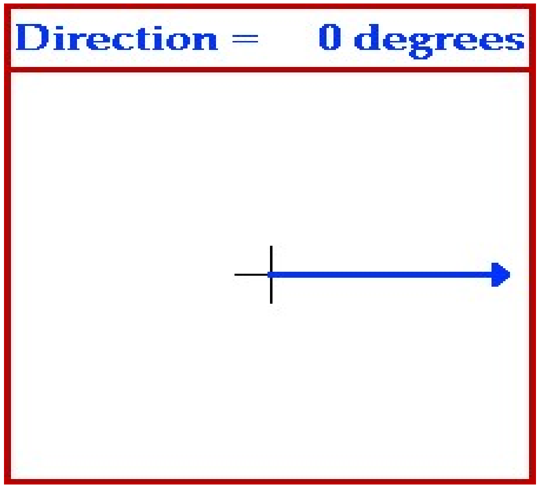
Some instruments for measuring scalar quantities

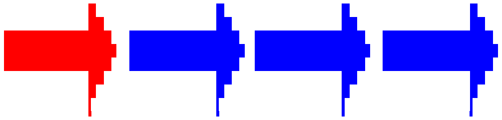




2-2 Physical quantities
Vector Quantities

- Vector quantities need both <u>magnitude</u> (size) and <u>direction</u> to completely describe them.
- Examples: force, displacement, velocity….
- A vector quantity is specified by
  1- a value with appropriate unit (a magnitude)
  2- a direction.
- Example: Wind's velocity is 3 m/s towards east.





2-2 Physical quantities
Vector Quantities

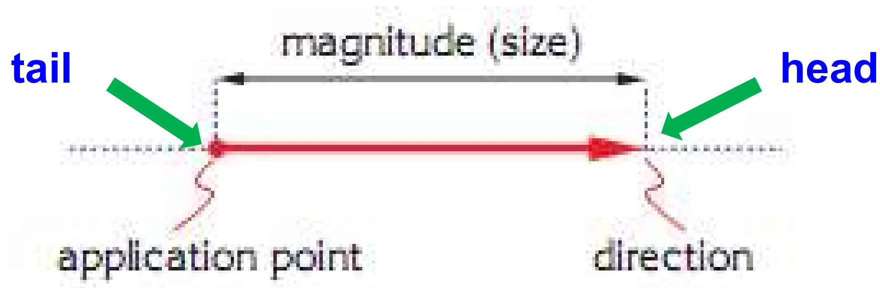
Look at the planes below. They are flying with the same speed!
Do you think that the direction is important?

What is the difference between velocity and speed?

2-2 Physical quantities

Notations-Vector Quantities

On a graph, a vector quantity is drawn as an arrow.



The length of the arrow is proportional to the magnitude of the vector quantity. Every vector has an application point and direction.
The head of the arrow shows the direction of the vector.

A vector quantity is denoted by an arrow placed over its symbol.

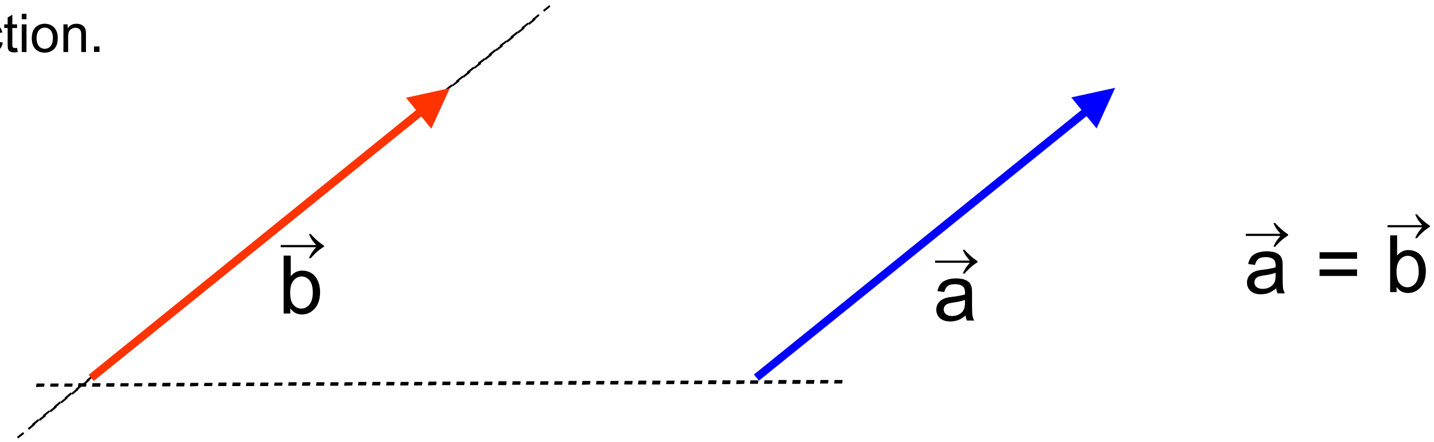
$$\vec{a}$$

The magnitude (absolute value) of a vector quantity is indicated by its symbol without an arrow.

$$|\vec{a}| = a$$

2-3 Adding Vectors Geometrically
Equality of two vectors

Two vectors are equal if they have the same magnitude and same direction.

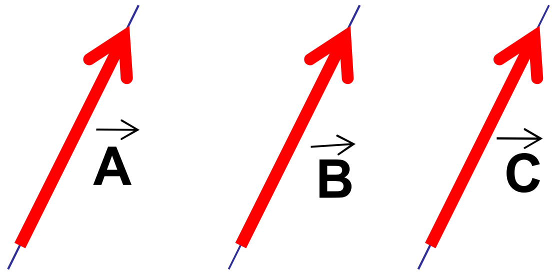


In a diagram, a vector may be moved to a new position without changing its magnitude and direction.

This property allows us to translate a vector parallel to itself in a diagram without affecting the vector.

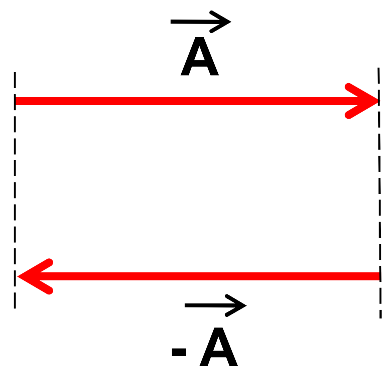
2-3 Adding Vectors Geometrically
Equal and negative vectors

Recall that if two vectors have the same magnitude and direction, they are called equal vectors.



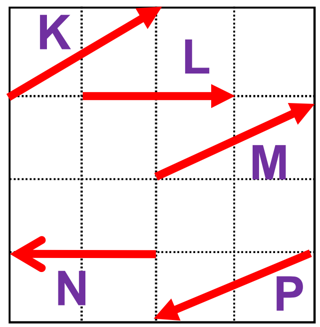
$$\vec{A} = \vec{B} = \vec{C}$$

If two vectors have the same magnitude but opposite direction they are called negative vectors.

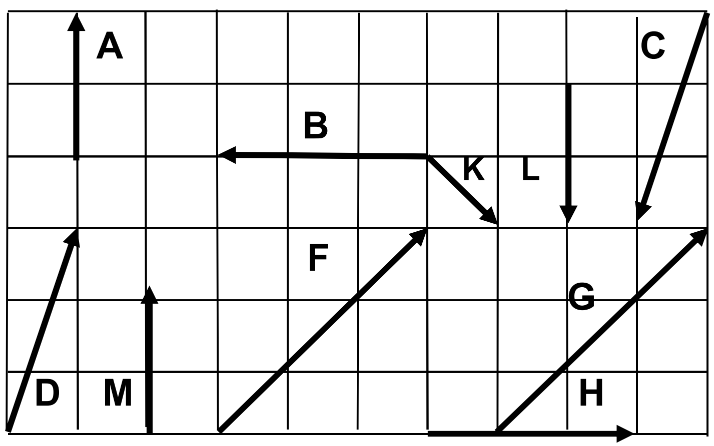


2-3 Adding Vectors Geometrically
Equal and negative vectors
Example 6

According to given figure below which vectors are equal, which vectors are negative?



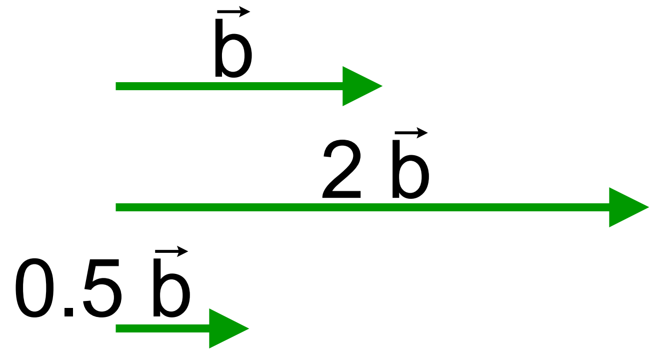
According to given figure below which vectors are equal, which vectors are negative?



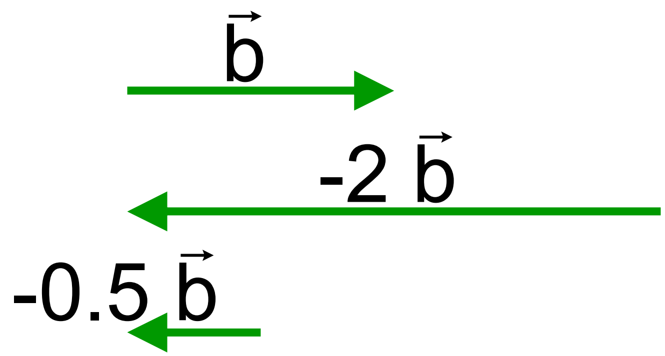
2-3 Adding Vectors Geometrically
Multiplying a vector by a scalar

The magnitude of the vector can be multiplied or divided by a scalar.

If the scalar is positive, the direction of the result is the same as of the original vector.

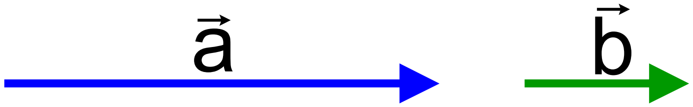


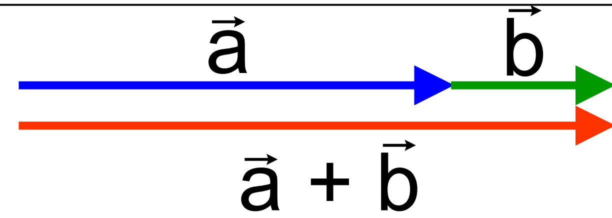
If the scalar is negative, the direction of the result is opposite that of the original vector.



2-3 Adding Vectors Geometrically

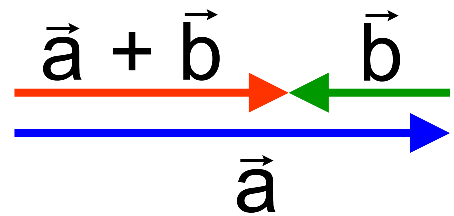
If two vectors have the same direction, the magnitude of the sum of two vectors equal to the sum of their magnitudes





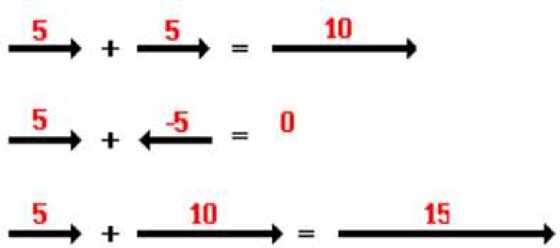
$$\left|\vec{a} + \vec{b}\right| = a + b$$

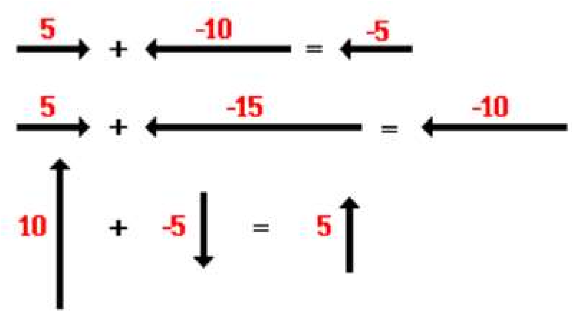
If two vectors have the opposite directions the magnitude of the sum of two vectors equal to the difference of their magnitudes.



$$\left|\vec{a} + \vec{b}\right| = a - b$$

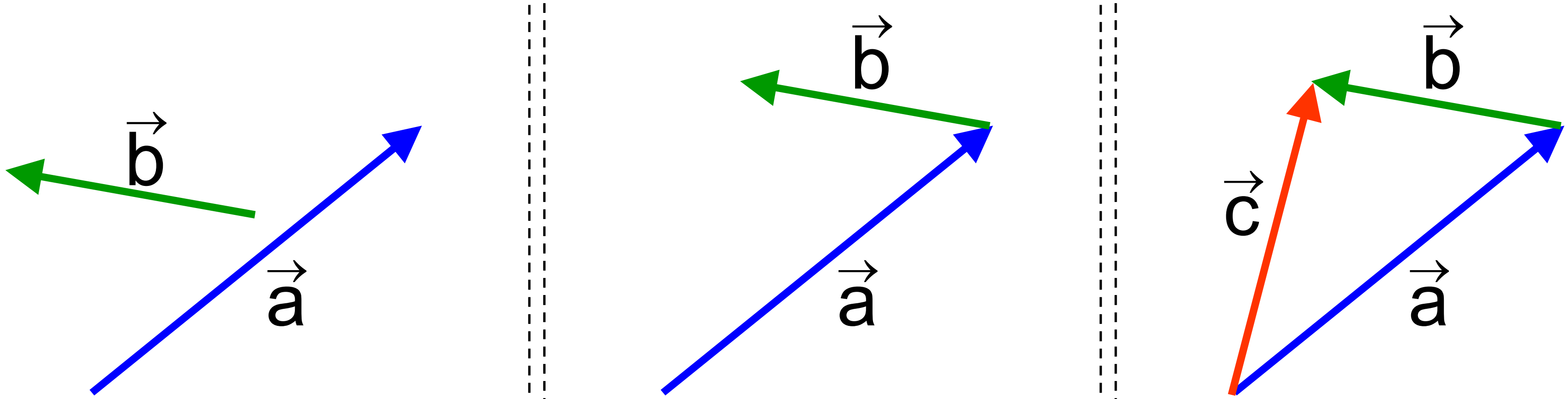
Example





2-3 Adding Vectors Geometrically
Adding two vectors

$$\vec{a} + \vec{b} = \vec{c}$$



Shift vector $\vec{b}$ so that its tail is at the head of vector $\vec{a}$.

The vector sum $\vec{c}$ is the vector drawn from the tail of $\vec{a}$ to the head of $\vec{b}$.

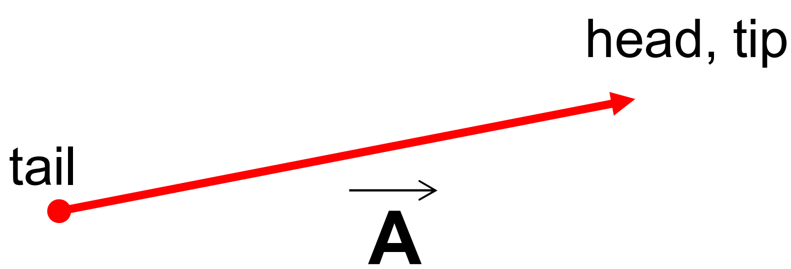
2-3 Adding Vectors Geometrically

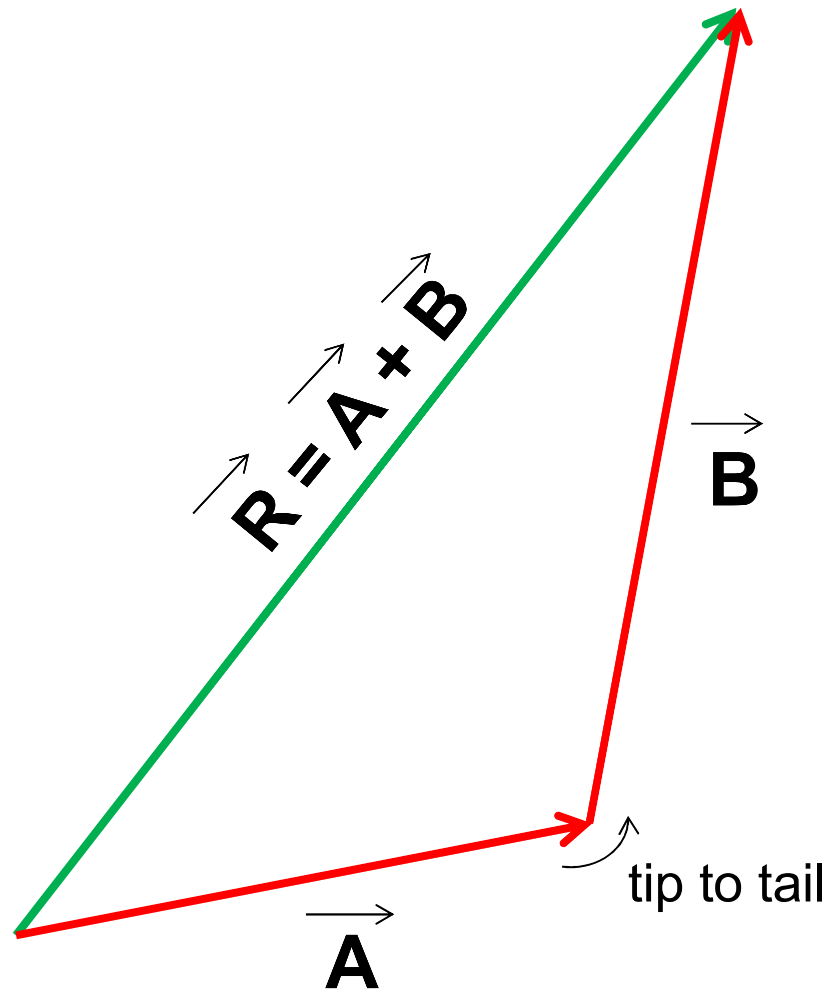
Adding two vectors

Graphical Method

This is also called "tip-to-tail". vector Addition. To add vectors **A** and **B**, the tip of vector **A** is placed on the tail of vector **B**. When we draw a straight line from the tail of the first vector to the tip of the last vector, we draw the resultant vector.

The resultant is drawn from the origin of A to the end of the last vector.



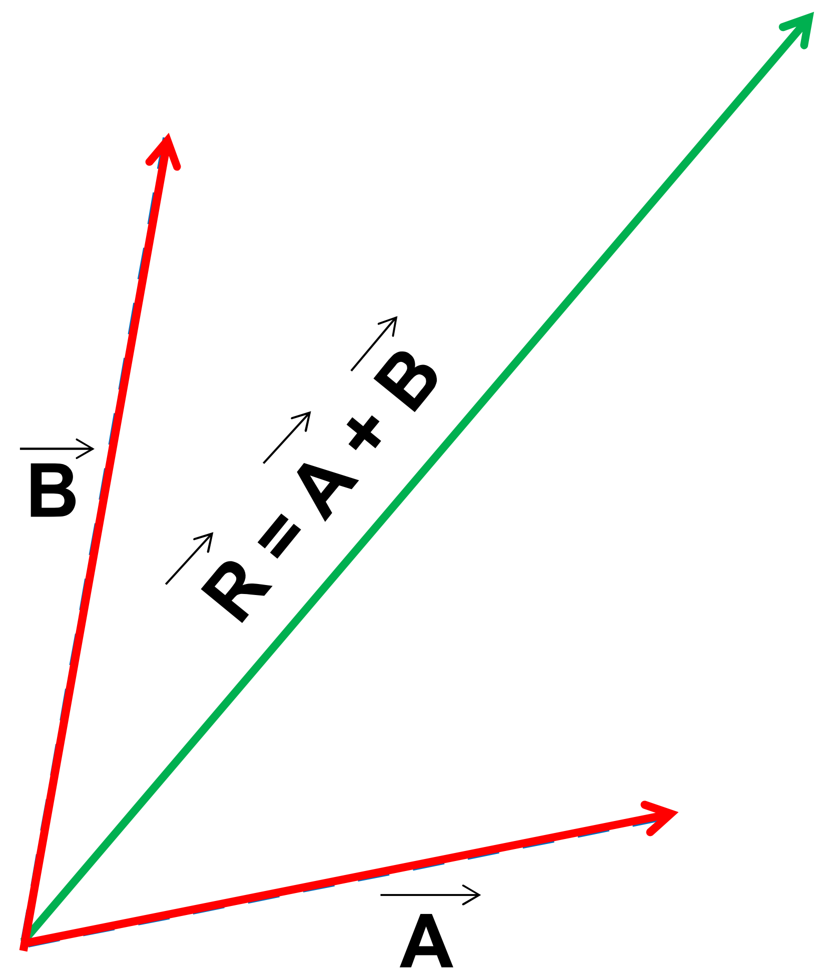


2-3 Adding Vectors Geometrically
Adding two vectors
Alternative Graphical Method

When you have only two vectors, you may use the **Parallelogram Method**

All vectors, including the resultant, are drawn from a common origin.

The remaining sides of the parallelogram are sketched to determine the diagonal, **R.**

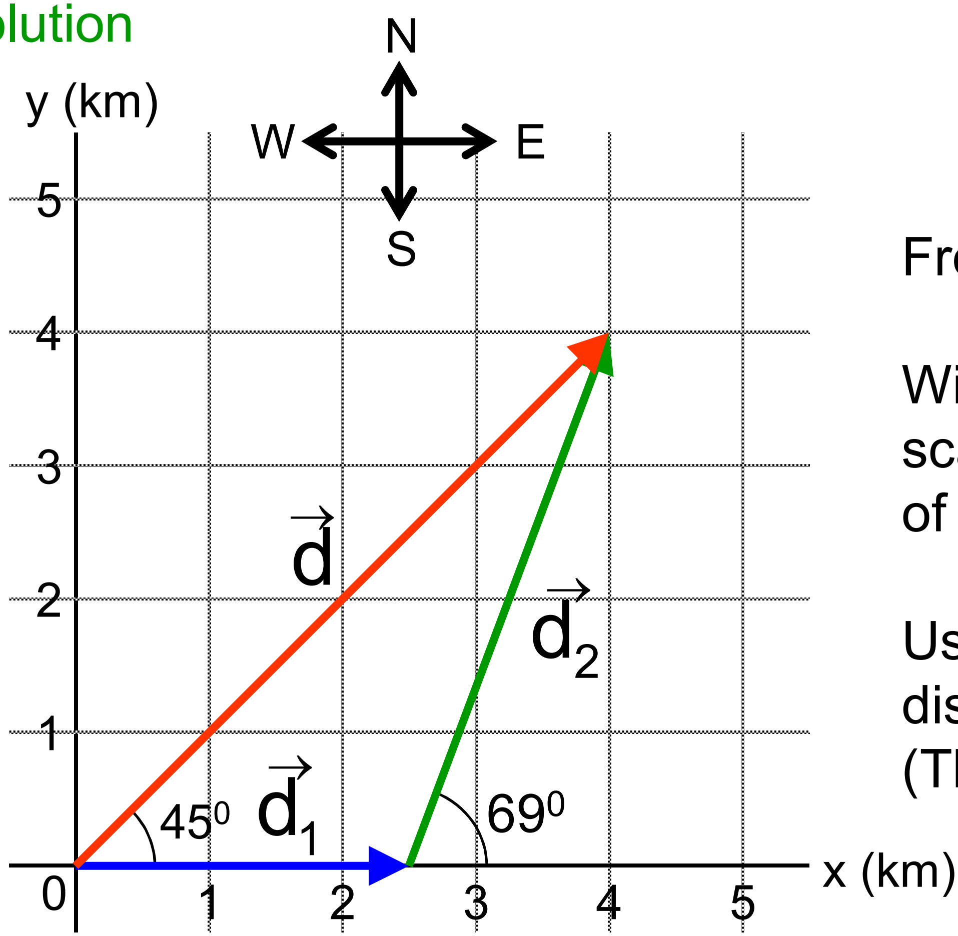


2-3 Adding Vectors Geometrically

Example 7

A man walks due east for a distance of 2.50 km.
Then he walks in a direction 69^0 north of east a distance of 4.27 km.
What is his total displacement?

Solution



$$\vec{d} = \vec{d}_1 + \vec{d}_2$$

From the Graph

With a ruler and using the proper scale of the figure, the magnitude of the total displacement = 5.7 km

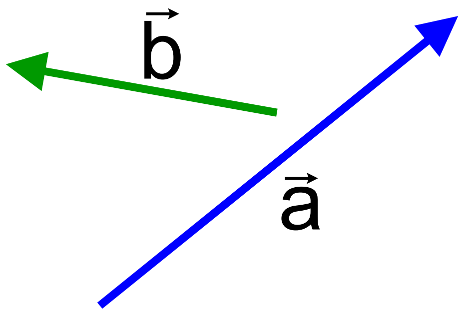
Using a protractor, the total displacement is 45^0 north of east. (This also clear from the figure.)

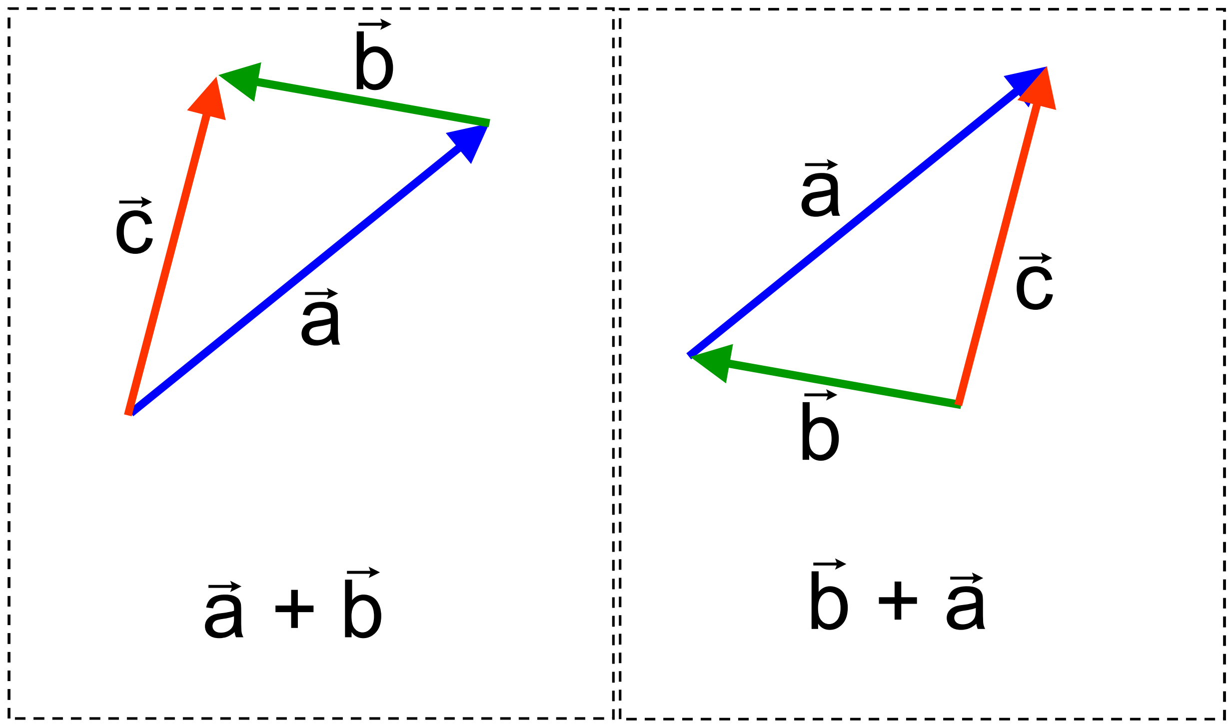
2-3 Adding Vectors Geometrically
Commutative law

Commutative law

The order in which the vectors are added doesn't affect the result.

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

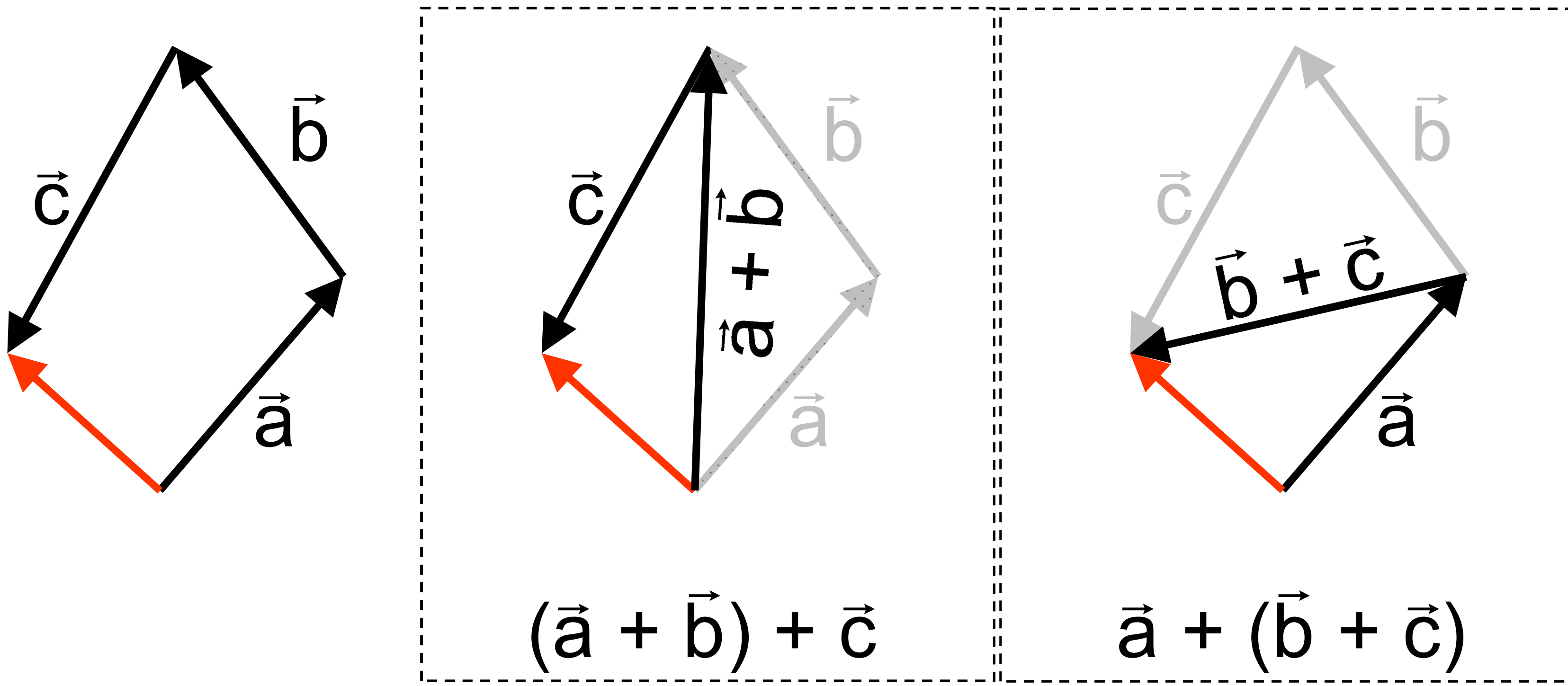




2-3 Adding Vectors Geometrically
Associative law

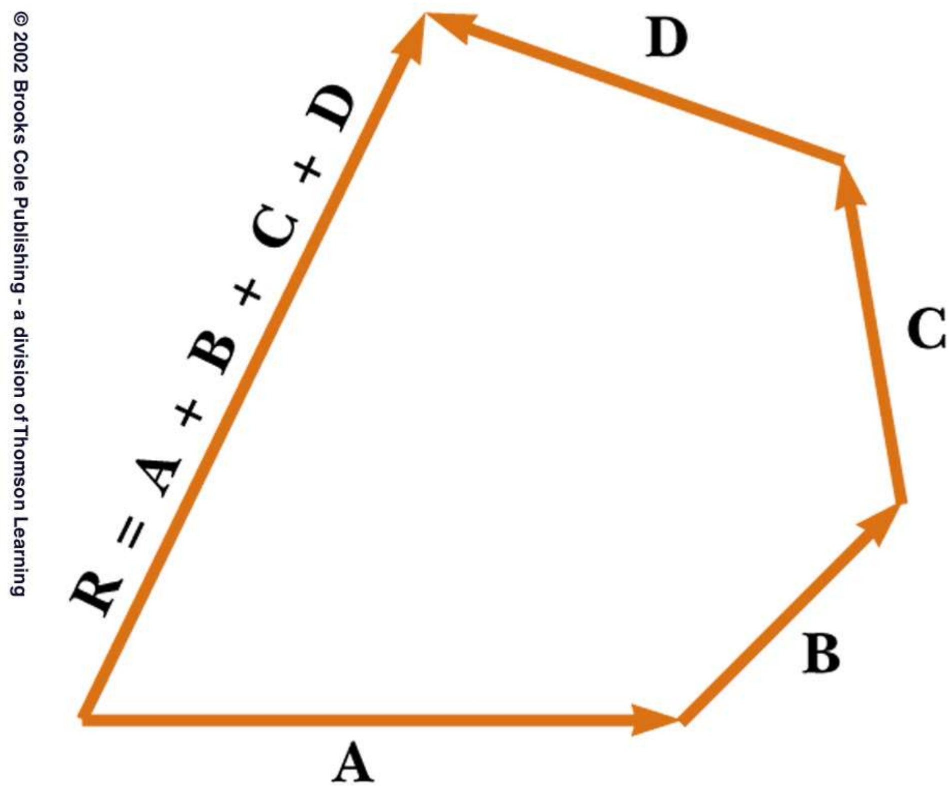
When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. This is called the associative law of addition.

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

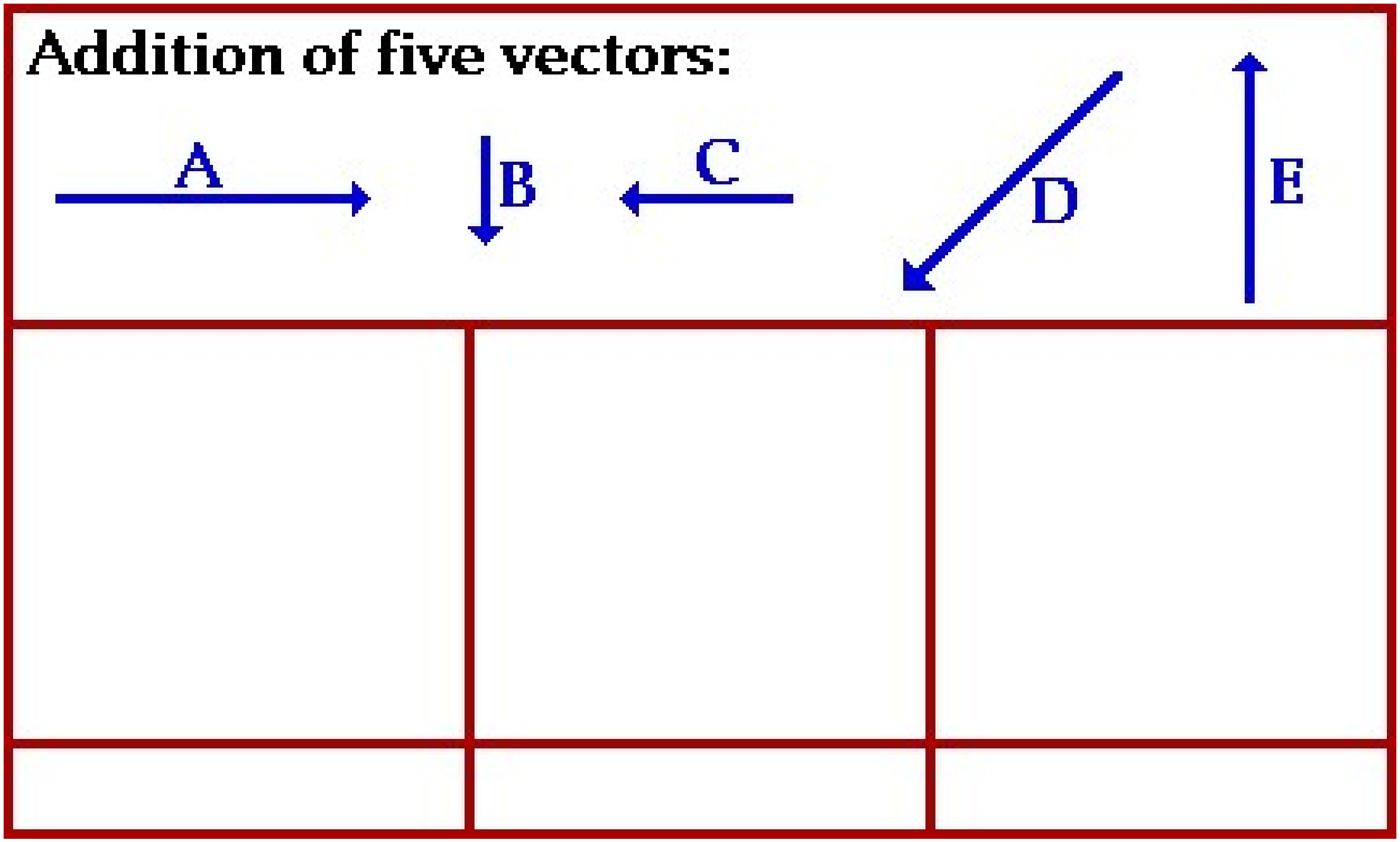


2-3 Adding Vectors Geometrically
Associative law

When you add more than two vectors, the resultant is the line draw from the origin of the first vector to the end of the last vector.



2-3 Adding Vectors Geometrically
Associative law

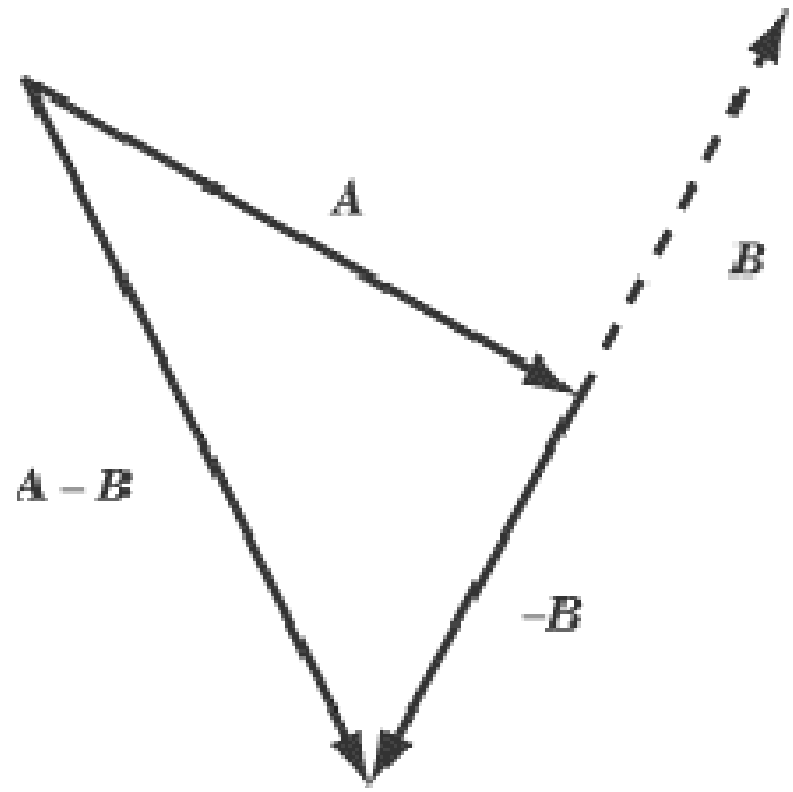


2-3 Adding Vectors Geometrically
Subtracting vectors
Special case of vector addition

If $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, then use $\vec{\mathbf{A}} + (-\vec{\mathbf{B}})$

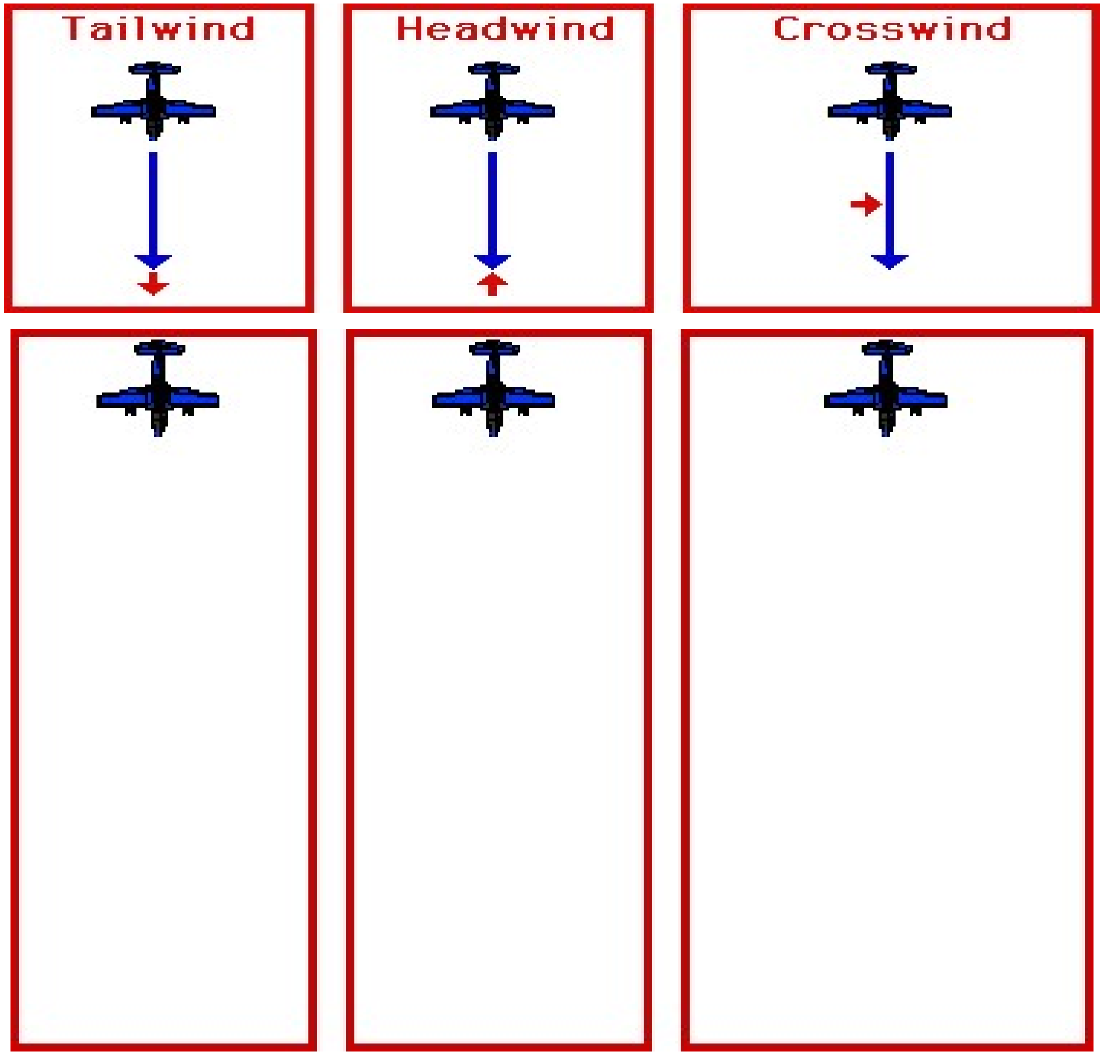
To subtract B from A, take a vector of the same magnitude as B, but pointing in the opposite direction, and add that vector to A, using either the tip-to-tail method or the parallelogram method.

The vectors **B & – B** have the same magnitude but point in opposite directions.
B + (-B) = 0

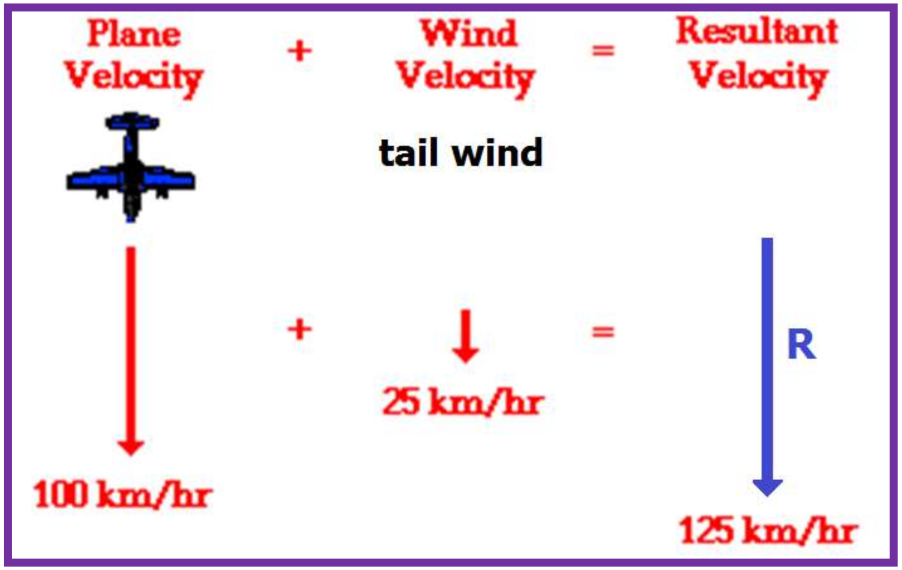


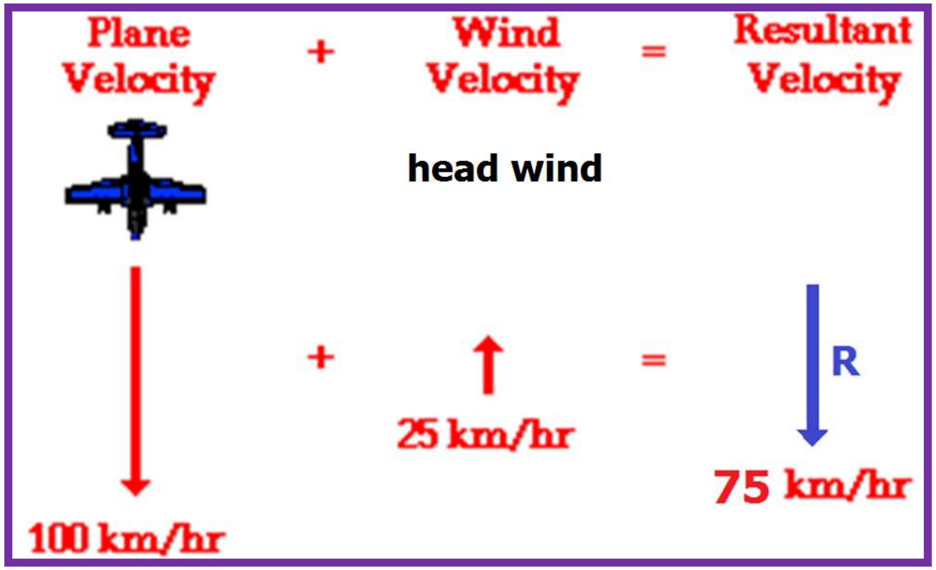
2-3 Adding Vectors Geometrically

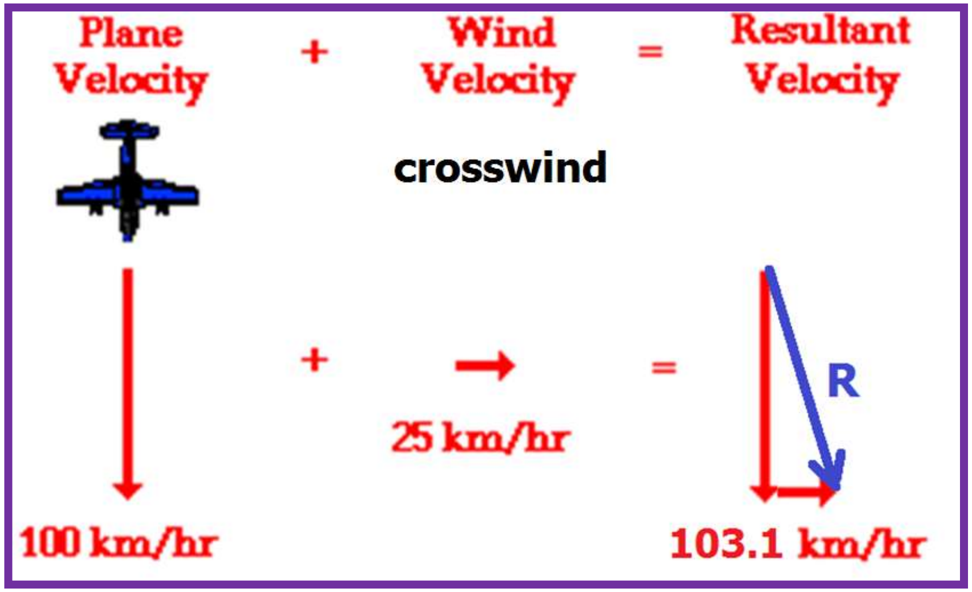
Air planes and wind directions



2-3 Adding Vectors Geometrically
Example 8



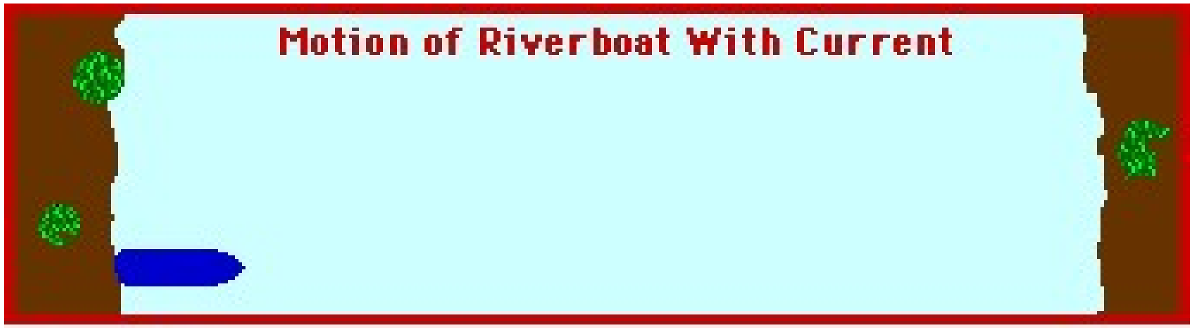


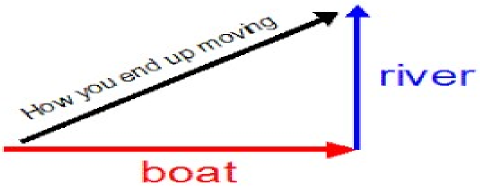


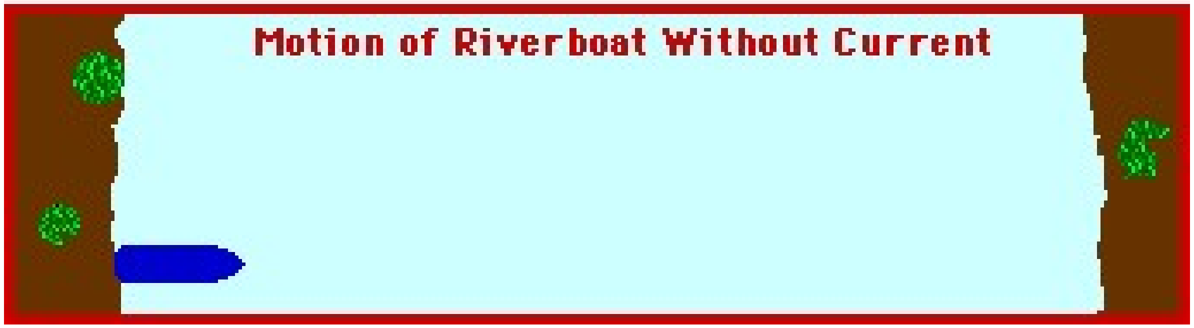
2-3 Adding Vectors Geometrically

Example 9

Draw the direction of boats.

1) 



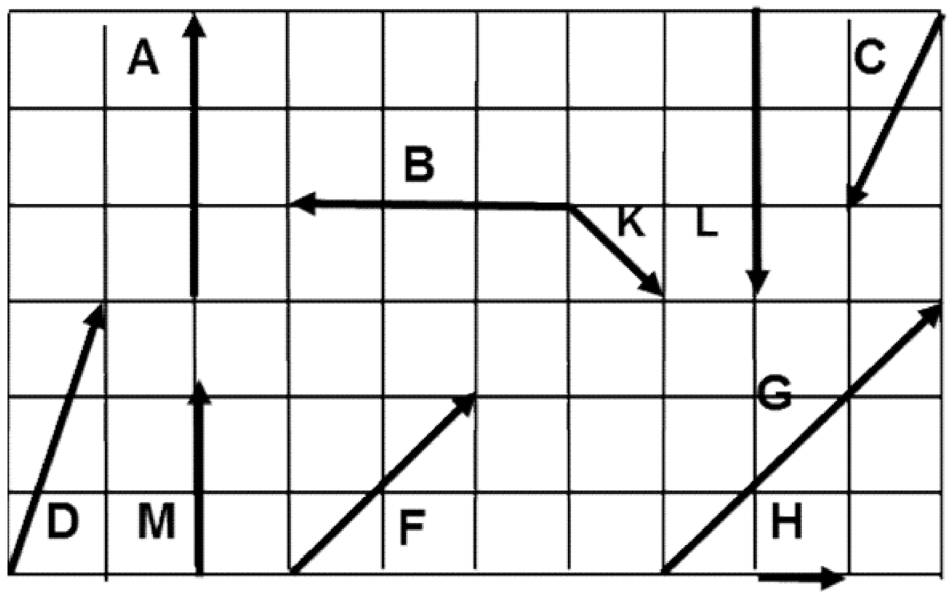
2) 

2-3 Adding Vectors Geometrically

Example 10

Look at the table and find the equivalent vector for the following equations:

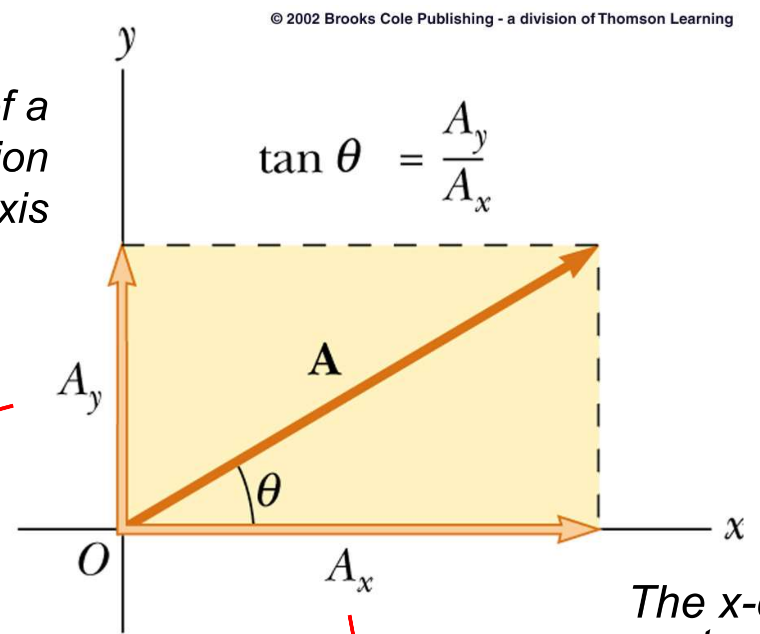
1) **A - B =**
2) A + H =
3) F + C =
4) G – D =
5) K + D =
6) L – K =
7) B + L =
8) M + C =
9) G + L =
10) A + K + H =
11) K + C + A =
12) G – 2H =
13) K + C + D =
14) G + A – H =



2-4 Components of Vectors
Projecting a vector on an axis

- The projections of a vector on the x and y axis are called the components of the vector.

The y-component of a vector is the projection along the y-axis



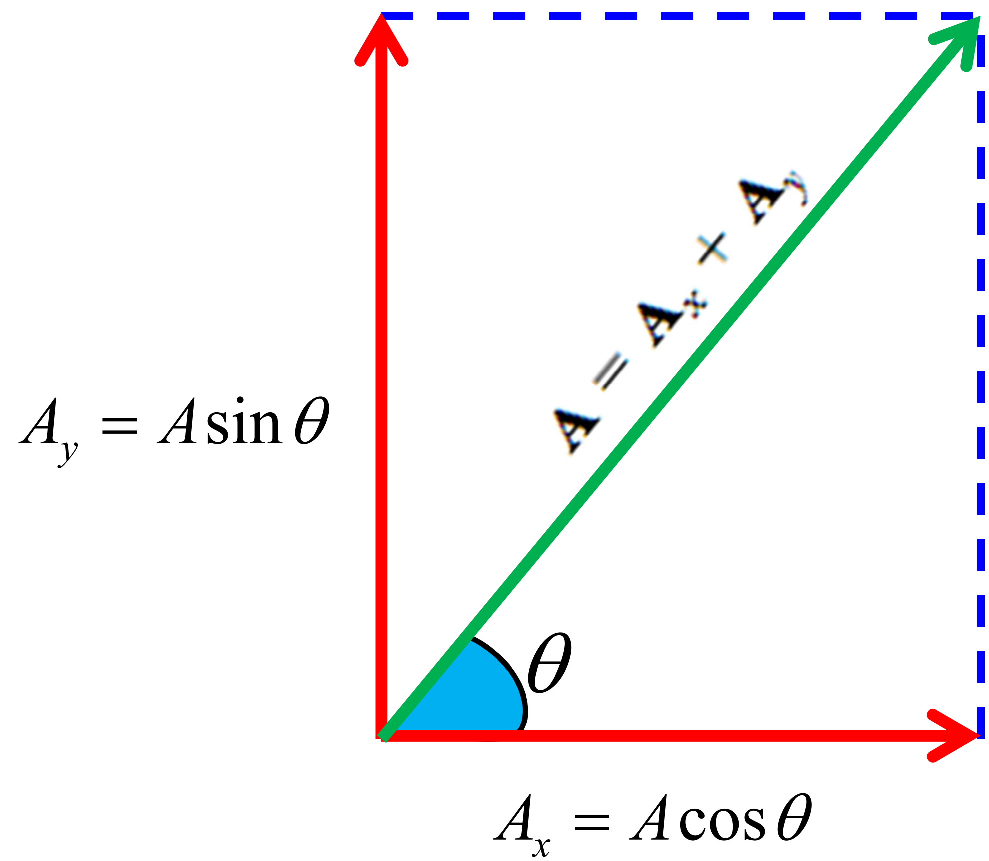
$$A_y = A\sin\theta$$

The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

2-4 Components of Vectors

Projecting a vector on an axis

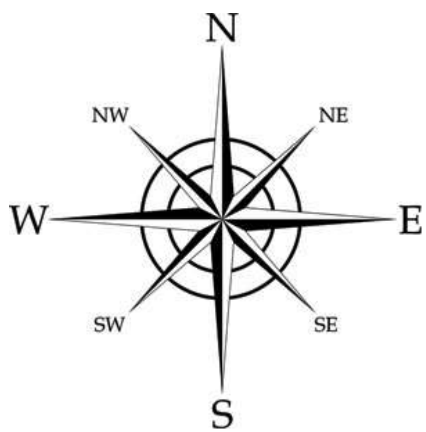


2-4 Components of Vectors

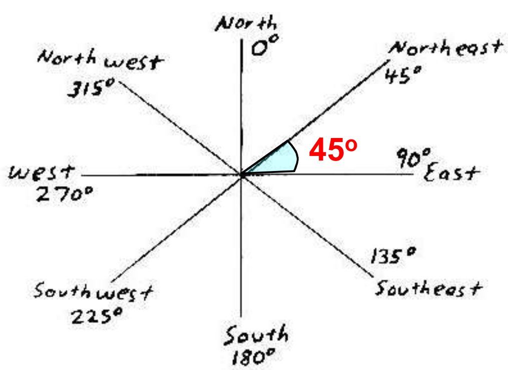
Directions

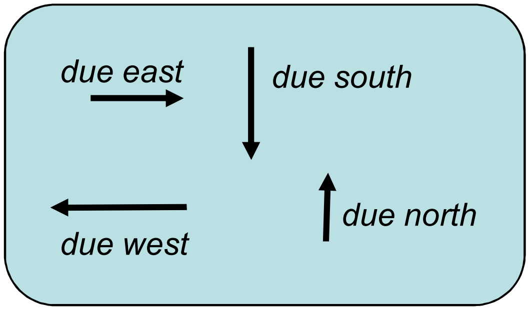
Cardinal directions: There are cardinal directions north, east, south, and west, commonly denoted by their initials N, E, S, and W.

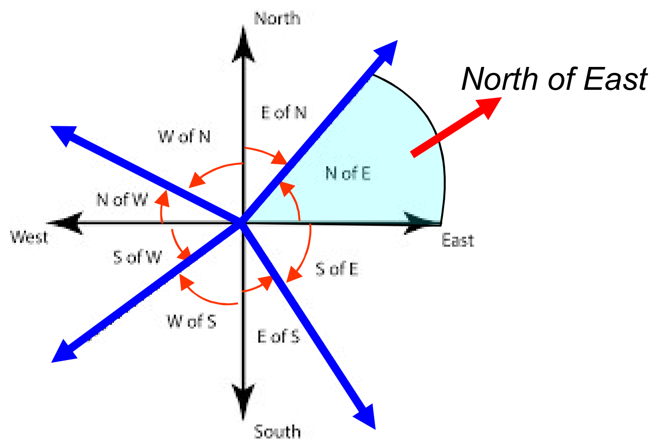
Intermediate directions: The intermediate directions are northeast (NE), southeast (SE), southwest (SW), and northwest (NW). Northeast (or NE) means exactly halfway between North and East or 45° N of E or 45° E of N. This is also true of Southeast (SE), Southwest (SW), Northeast(NE) and Northwest (NW).



Windrose







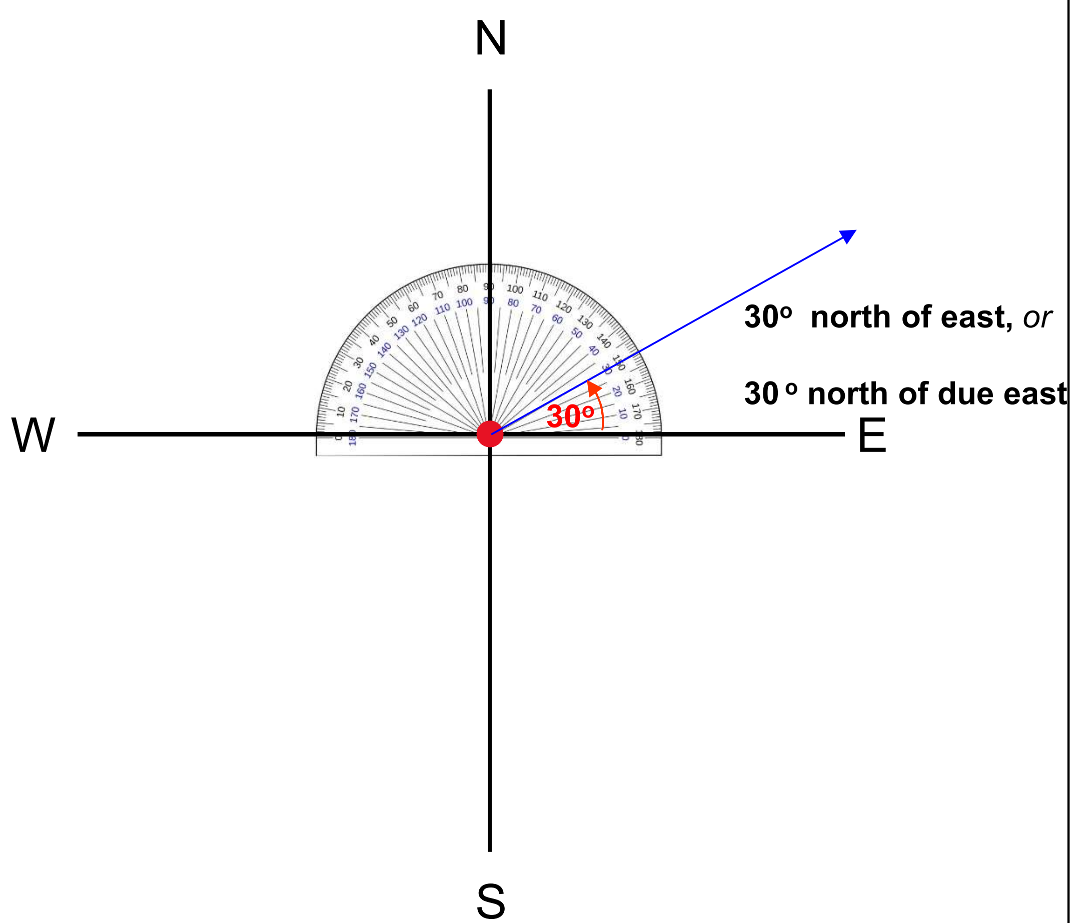
North of East (N of E) can described as North "from" East, because it means that the angle is measured Northward from the Eastward direction.

2-4 Components of Vectors

Example 11

Draw the following angles:

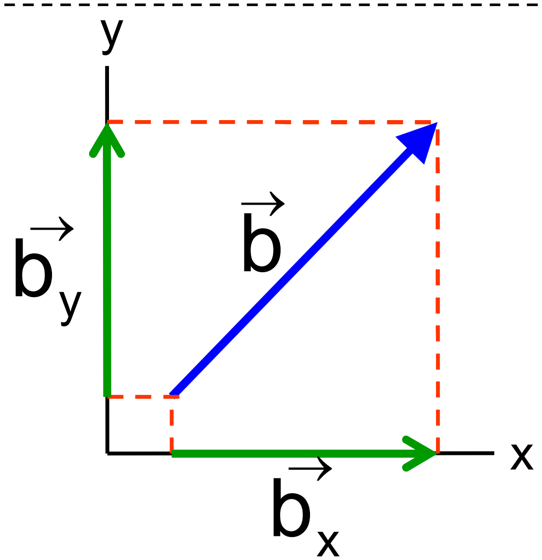
1. 45° North of East
2. 15° East of North
3. 65° South of East
4. 18° South of West
5. 60° West of North
6. 30° North of East
7. 55° South of East
8. 25° West of South
9. 60° East of due North
10. 20° West of due South
11. 50° East of due South
12. 55° South of due East
13. North-East
14. South-West
15. North-West
16. South-Wast



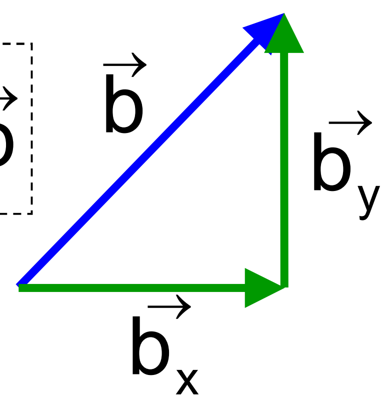
2-4 Components of Vectors

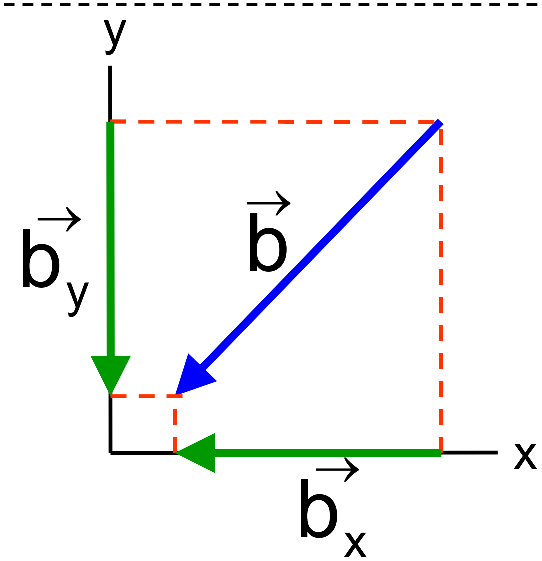
Projecting a vector on an axis

To find the projection of a vector along an axis,
draw **perpendicular** lines from the two ends of the vector to the axis.

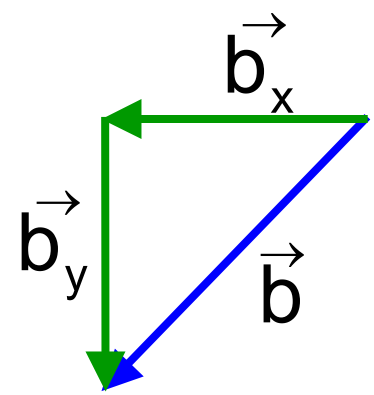


$$\vec{b}_x + \vec{b}_y = \vec{b}$$





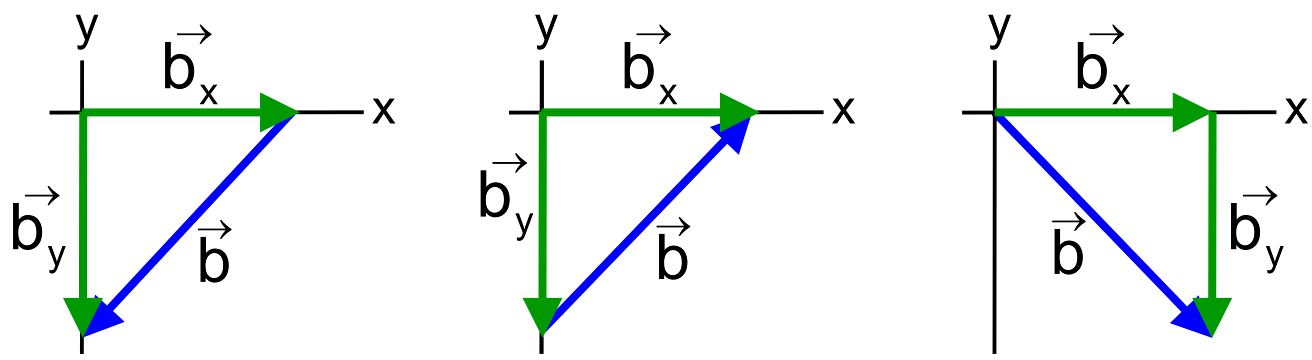
$$\vec{b}_x + \vec{b}_y = \vec{b}$$



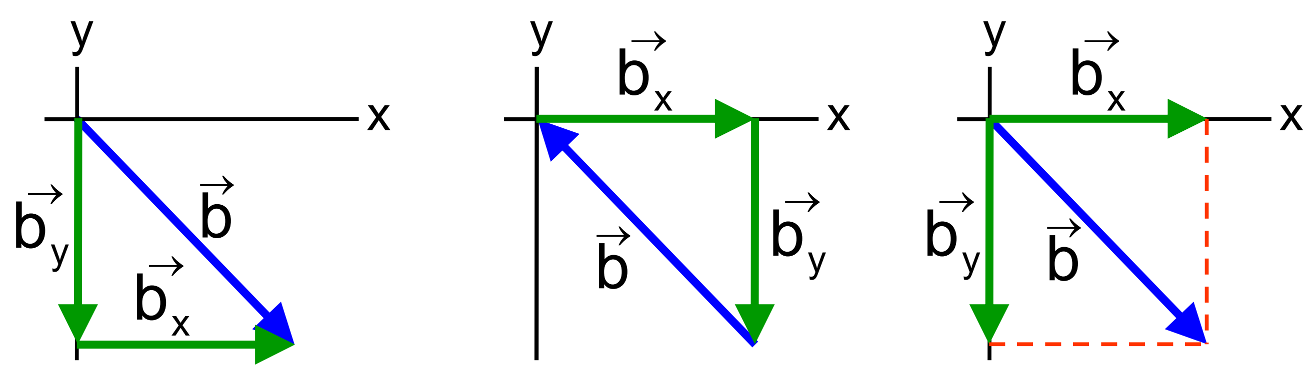
The projection has the same direction along an axis as the vector.

2-4 Components of Vectors
Example 12

Indicate the correct projections.



Solution	Wrong	Wrong	Correct

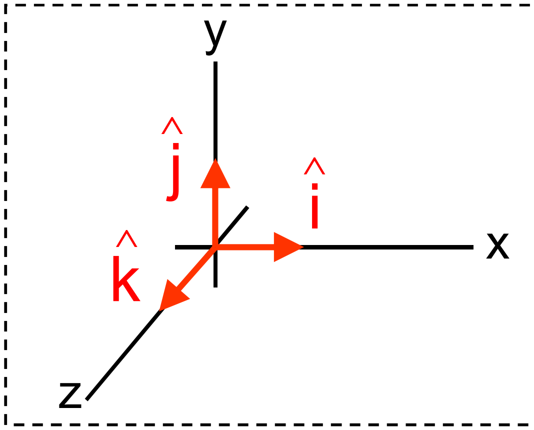


Solution	Correct	Wrong	Correct

2-4 Components of Vectors

Unit vectors

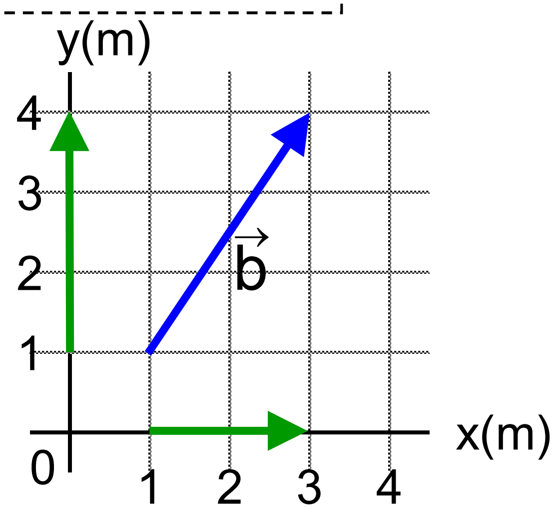
A **unit vector** is a vector used to specify a direction.
It has a magnitude of one.
It has no dimension and thus has no unit.



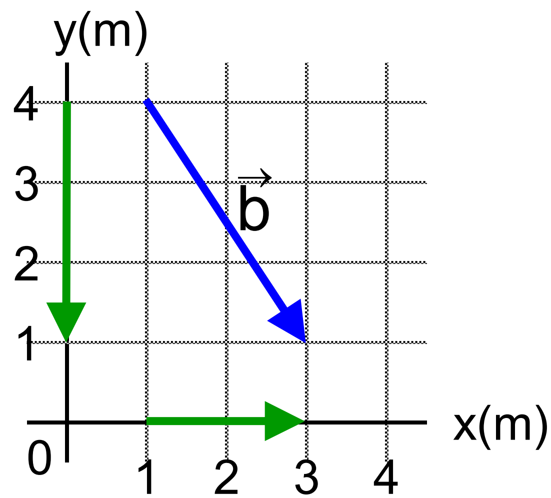
$\hat{i}$ is a unit vector pointing in the positive x direction.

$\hat{j}$ is a unit vector pointing in the positive y direction.

$\hat{k}$ is a unit vector pointing in the positive z direction.



$\vec{b} = (2.0\ \text{m})\,\hat{i} + (3.0\ \text{m})\,\hat{j}$



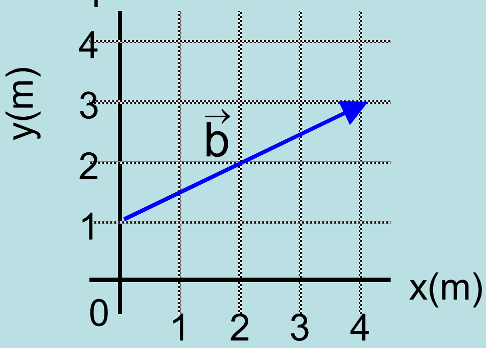
$\vec{b} = (2.0\ \text{m})\,\hat{i} + (-3.0\ \text{m})\,\hat{j}$

2-4 Components of Vectors

Example 13

Express the components of each vector in terms of unit vectors.

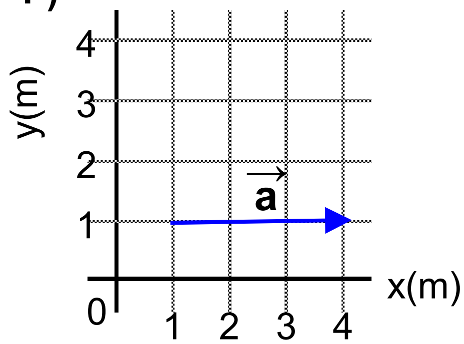
Example



Solution

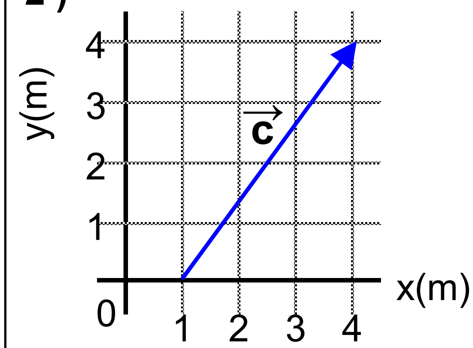
$\vec{b} = 4\,\hat{i} + 2\,\hat{j}$

1)



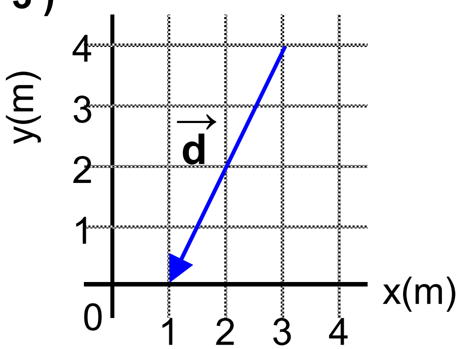
Solution

2)



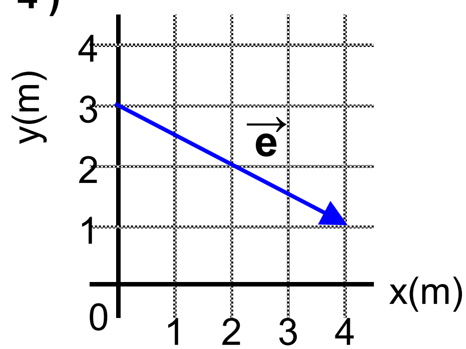
Solution

3)



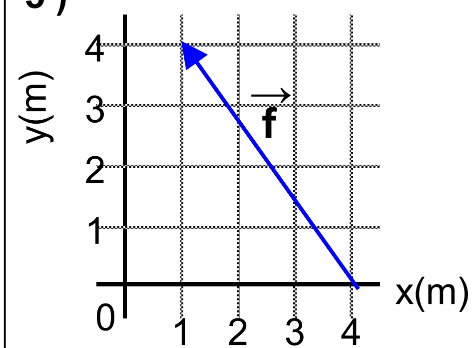
Solution

4)



Solution

5)



Solution

2-4 Components of Vectors

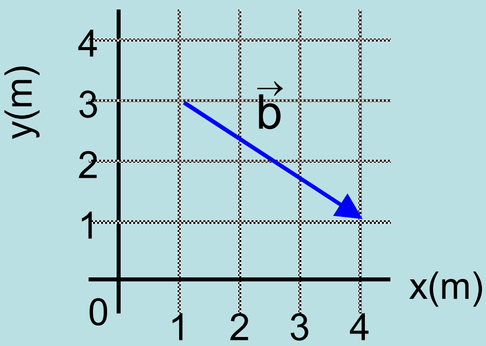
Example 14

Draw the vectors.

Example

$\vec{\mathbf{b}} = 3\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}}$

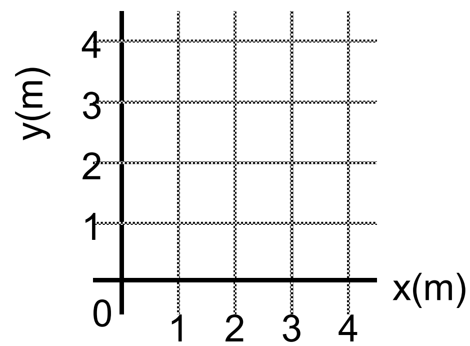
Solution



1)

$\vec{a} = 2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$

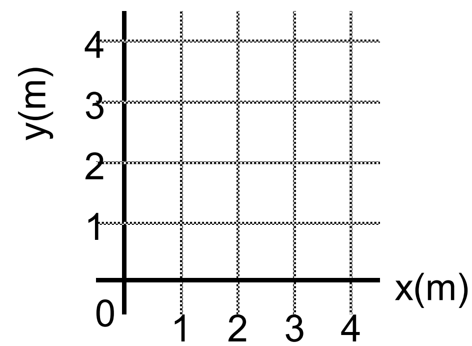
Solution



2)

$\vec{c} = -3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}$

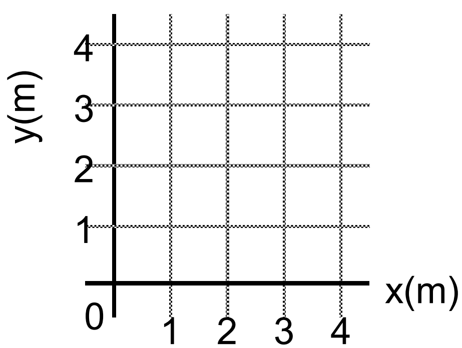
Solution



3)

$\vec{d} = -2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}$

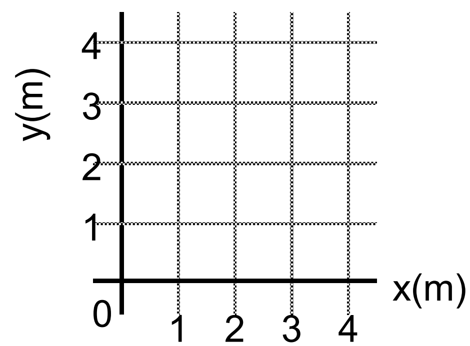
Solution



4)

$\vec{e} = 3\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

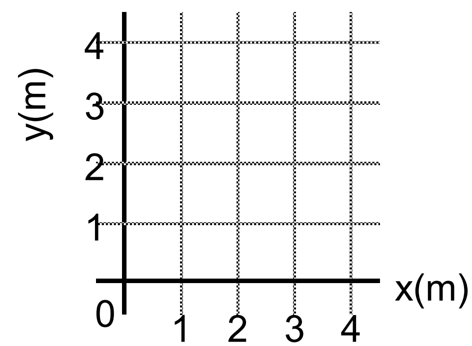
Solution



5)

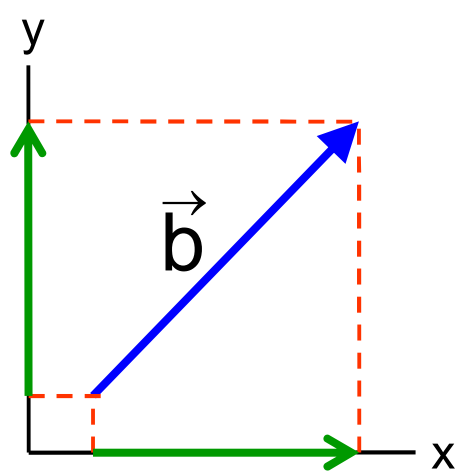
$\vec{f} = -\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$

Solution



2-4 Components of Vectors

Components of a vector



$$\vec{b}_y = b_y \hat{j}$$

b_y is the component of vector $\vec{b}$ on the y axis.

b_y is the y component of vector $\vec{b}$.

b_y can be negative.

$$\vec{b}_x = b_x \hat{i}$$

b_x is the component of vector $\vec{b}$ on the x axis.

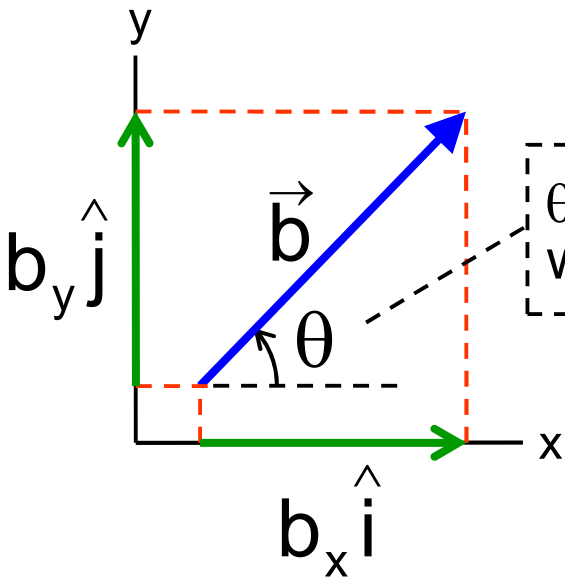
b_x is the x component of vector $\vec{b}$.

b_x can be negative.

$$\vec{b} = b_x \hat{i} + b_y \hat{j}$$

We resolve a vector by finding its components.

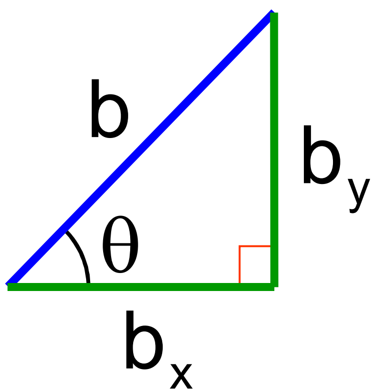
2-4 Components of Vectors
Finding components



θ is the angle that the vector $\vec{b}$ makes with the positive direction of the x axis.

$$b_x = b\cos\theta$$

$$b_y = b\sin\theta$$



$$b = \sqrt{b_x^2 + b_y^2}$$

$$\theta = \tan^{-1}\frac{b_y}{b_x}$$

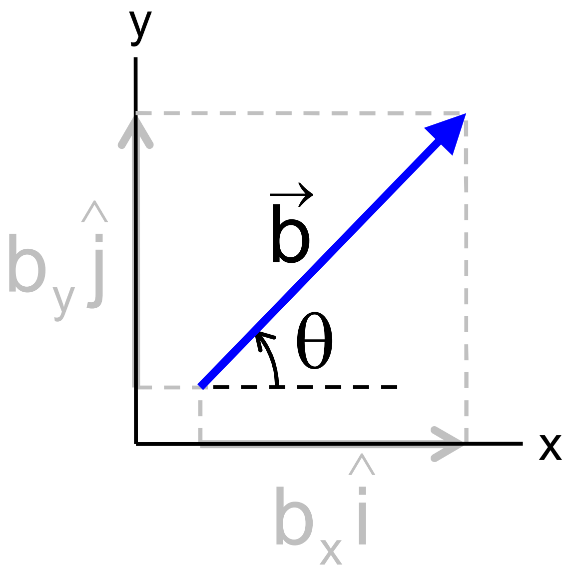
2-4 Components of Vectors

Specifying a vector

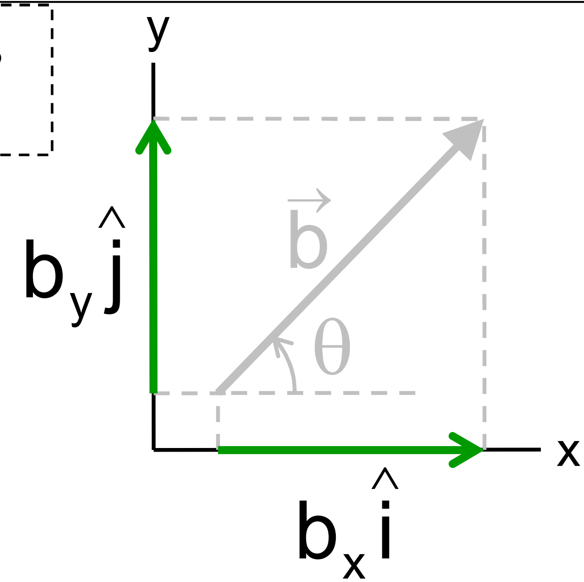
When working with a vector, you can use

its magnitude and direction | or | its components

Two dimensions
(a plane)



Magnitude and one angle
b and θ



Two components
x and y components

Three dimensions
(a space)

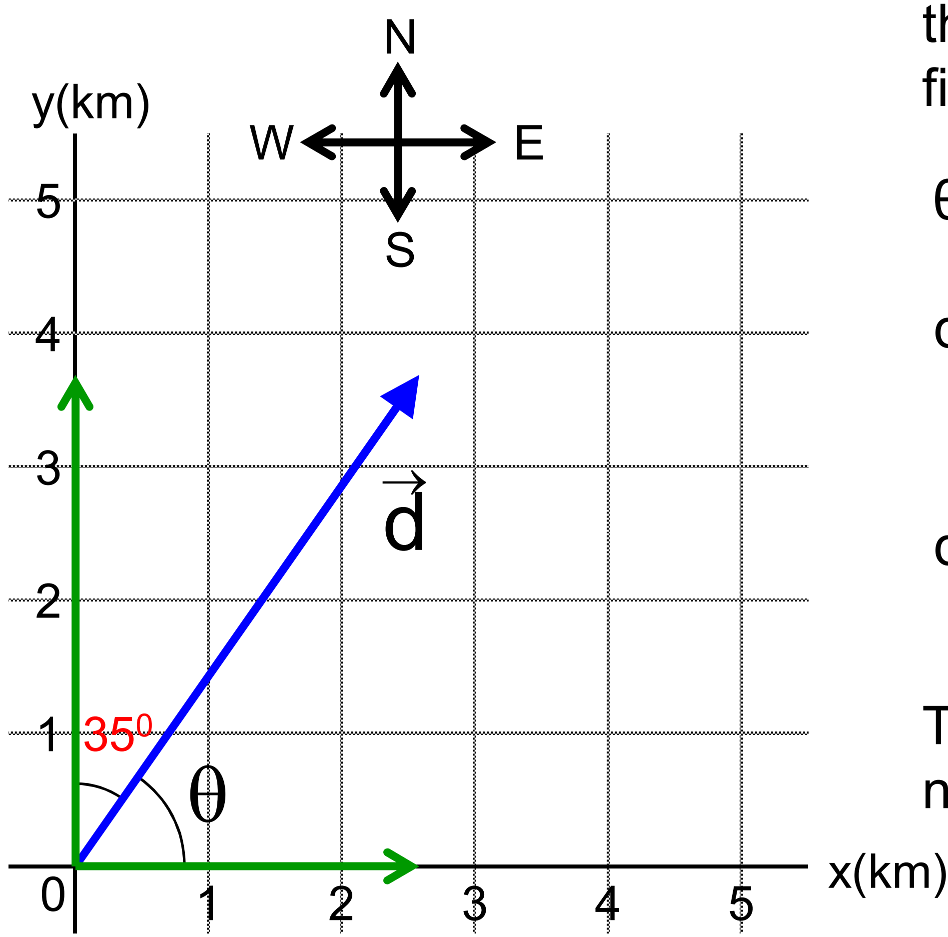
Magnitude and two angles
b, θ and ϕ

Three components
x, y, and z components

2-4 Components of Vectors
Example 15

A man walks 4.5 km in a direction making an angle of 35^0 east of due north. How far east and north is the man from his starting point?
Write the vector d with unit vectors.

Solution



We are given the magnitude and the angle of a vector and need to find the components of the vector.

$$\theta = 90° - 35° = 55°$$

$$d_x = d \cos\theta = (4.5\ \text{km})(\cos 55°)$$
$$= 2.6\ \text{km}$$

$$d_y = d \sin\theta = (4.5\ \text{km})(\sin 55°)$$
$$= 3.7\ \text{km}$$

The man is 2.6 km east and 3.7 km north of his starting point.

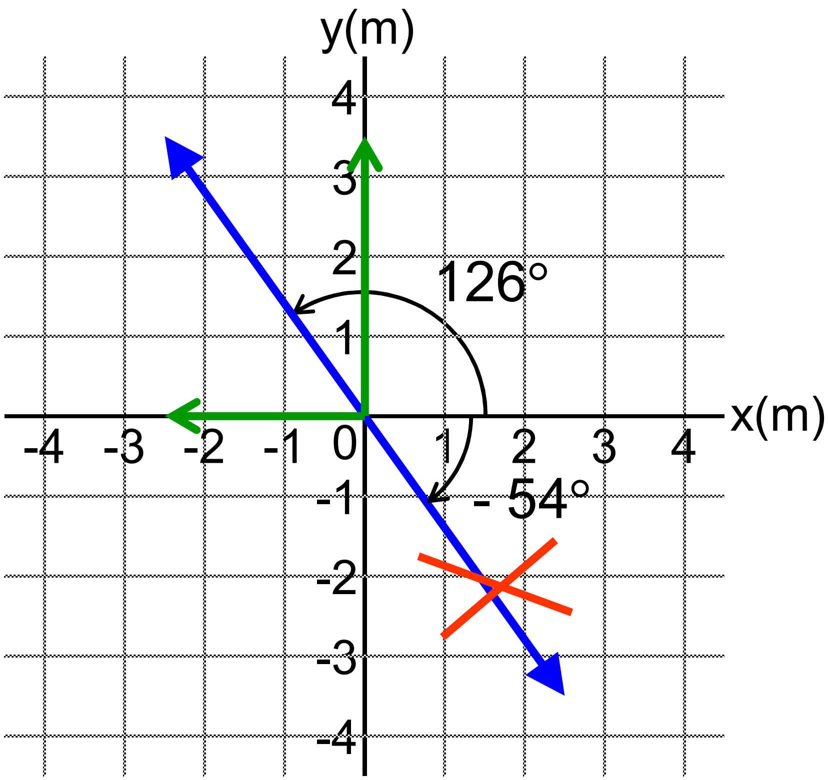
$$\vec{\mathbf{d}} = (2.6\ \text{km})\,\hat{\mathbf{i}} + (3.7\ \text{km})\,\hat{\mathbf{j}}$$

2-4 Components of Vectors
Example 16

Find the magnitude and direction of the following displacement vector

$$\vec{d} = (-2.5\ \text{m})\ \hat{i} + (3.5\ \text{m})\ \hat{j}$$

Solution



$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.5\ \text{m})^2 + (3.5\ \text{m})^2}$$

$$= 4.3\ \text{m}$$

Using a calculator

$$\theta = \tan^{-1}\frac{3.5\ \text{m}}{-2.5\ \text{m}} = -54°$$ ✗

This answer is not consistent with the directions of the components. The correct answer is

$$\theta = -54° + 180° = 126°$$

$$\tan\frac{3.5\ \text{m}}{-2.5\ \text{m}} = \tan\frac{-3.5\ \text{m}}{2.5\ \text{m}}$$

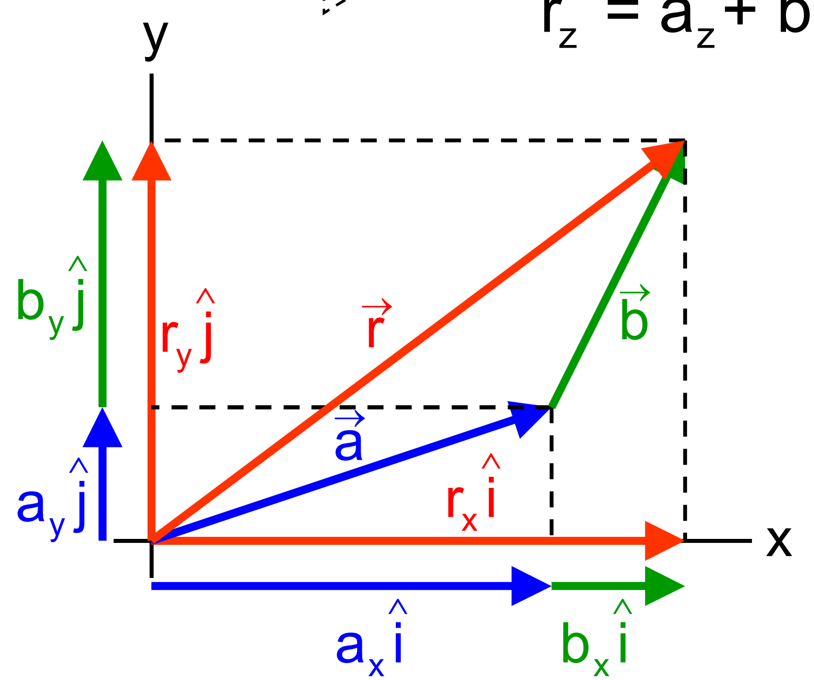
When taking the inverse of a trigonometric functions, always check the validity of your answer!

2-5 Adding Vectors by Components

$$\vec{r} = \vec{a} + \vec{b}$$

Two vectors are equal if their components are equal.

$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z$$



$$\vec{r} = \vec{a} - \vec{b}$$

$$r_x = a_x - b_x$$
$$r_y = a_y - b_y$$
$$r_z = a_z - b_z$$

2-5 Adding Vectors by Components
Example 17

- *An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid's height is 136 m and its width is 2.30×10^2 m. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?*

Solution

1. Define

   Given:

   Δy = 136 m

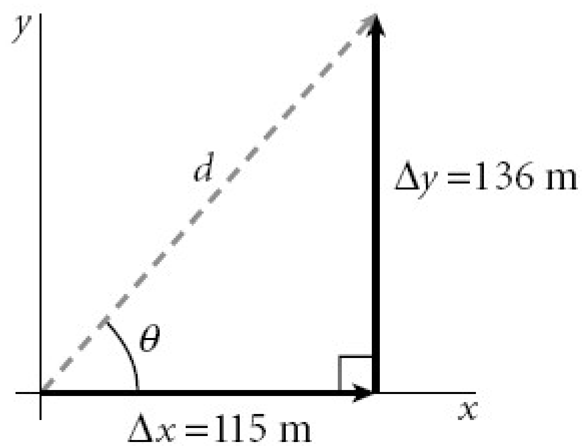
   Δx = 1/2(width) = 115 m

   Unknown:

   d = ? θ = ?

   Diagram:

   Choose the archaeologist's starting position as the origin of the coordinate system, as shown above.



2-5 Adding Vectors by Components

Example

Plan : Choose an equation or situation: The Pythagorean theorem can be used to find the magnitude of the archaeologist's displacement. The direction of the displacement can be found by using the inverse tangent function.

$$d = \sqrt{\Delta x^2 + \Delta y^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

$$d = \sqrt{\Delta x^2 + \Delta y^2} \qquad \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

$$d = \sqrt{(115 \text{ m})^2 + (136 \text{ m})^2} \qquad \theta = \tan^{-1}\left(\frac{136 \text{ m}}{115}\right)$$

$$d = 178 \text{ m} \qquad \theta = 49.8°$$

Evaluate:

Because *d* is the hypotenuse, the archaeologist's displacement should be less than the sum of the height and half of the width.

2-5 Adding Vectors by Components

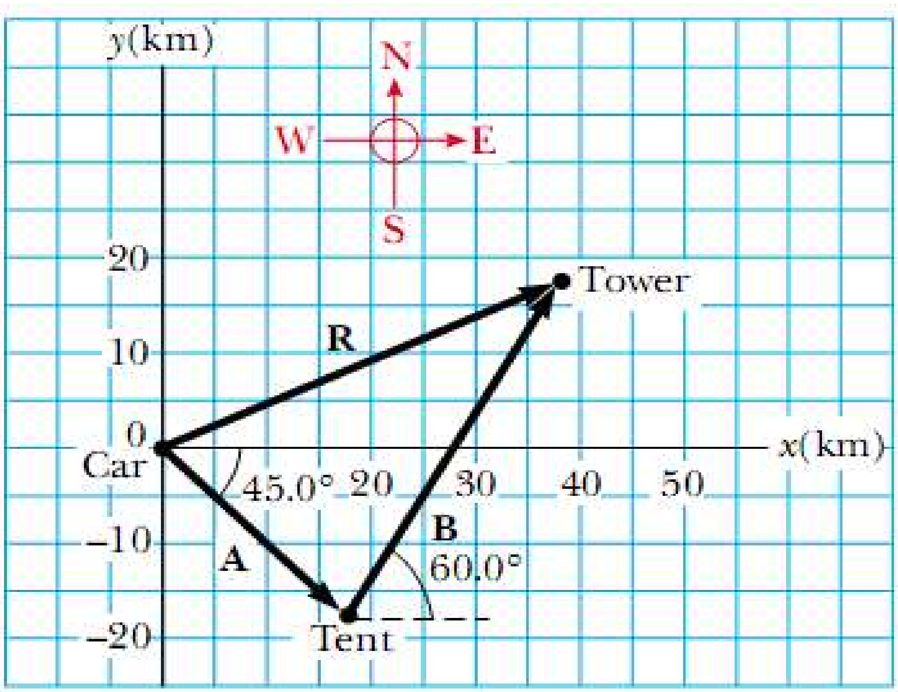
Example 18

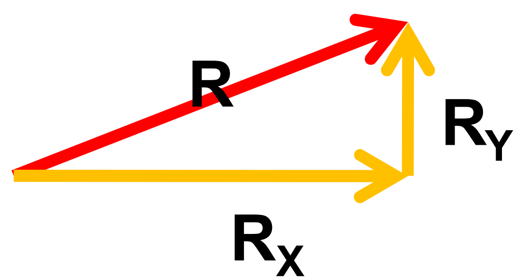
A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

(B) Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms

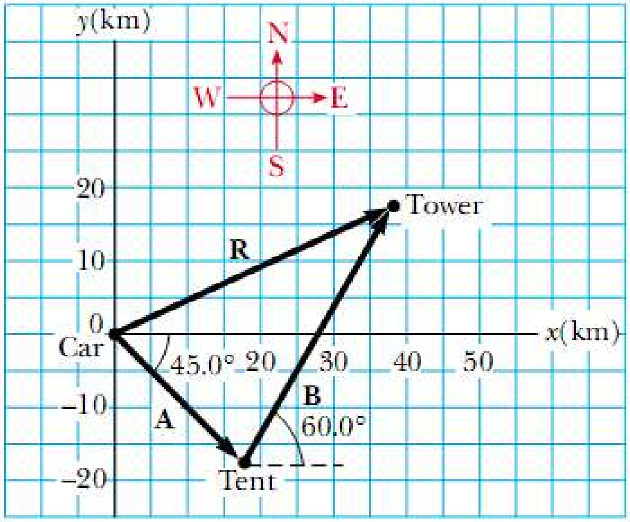
Solution





2-5 Adding Vectors by Components

(A) Determine the components of the hiker's displacement for each day.



$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = -17.7\text{ km}$$

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

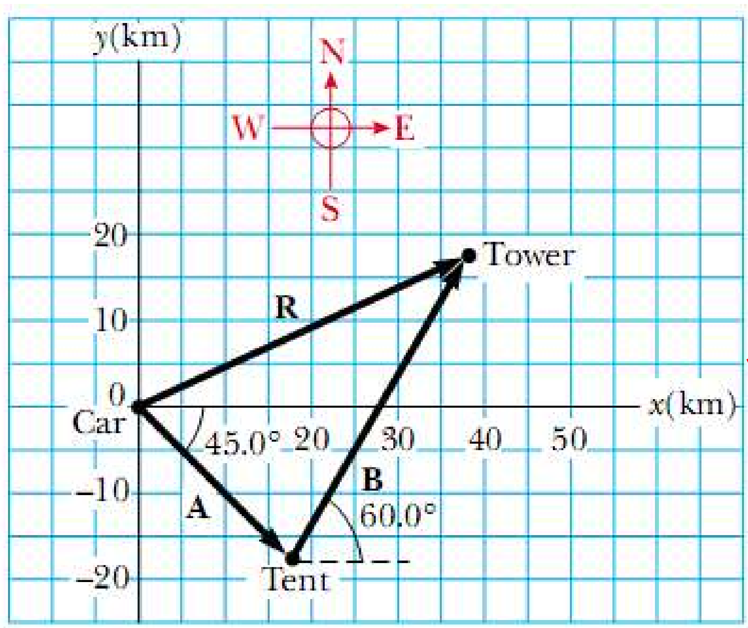
$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

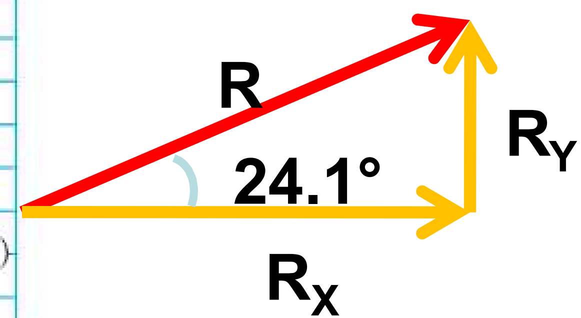
2-5 Adding Vectors by Components

(B) Determine the components of the hiker's resultant displacement R for the trip.

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$





$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

As a result it is found that the vector R has a magnitude of 41.3 km and is directed 24.1° north of east

2-5 Adding Vectors by Components

Questions

1.Find the magnitude and direction of the following displacement vector.

$$\vec{d} = (-2.5\ \text{m})\,\hat{i} + (3.5\ \text{m})\,\hat{j}$$

2. Three vectors a, b, and c have equal magnitudes of 20 m. Find the magnitude and direction of the sum of these vectors.