Chapter 1
Measurement

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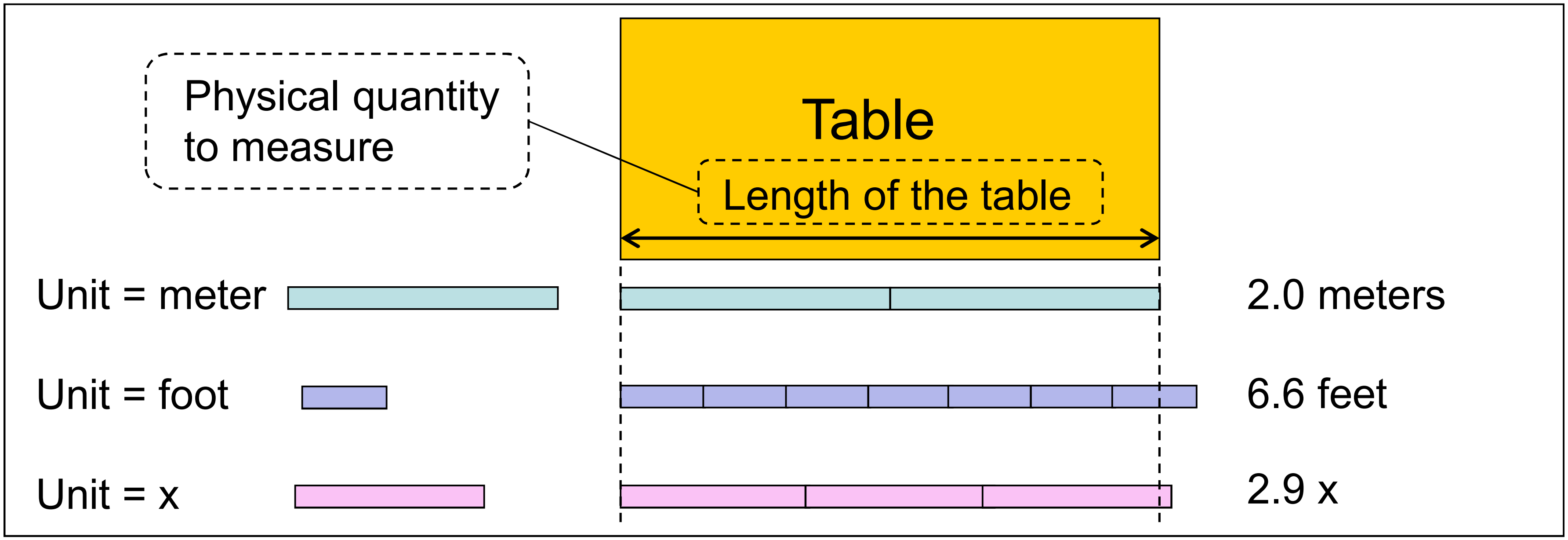
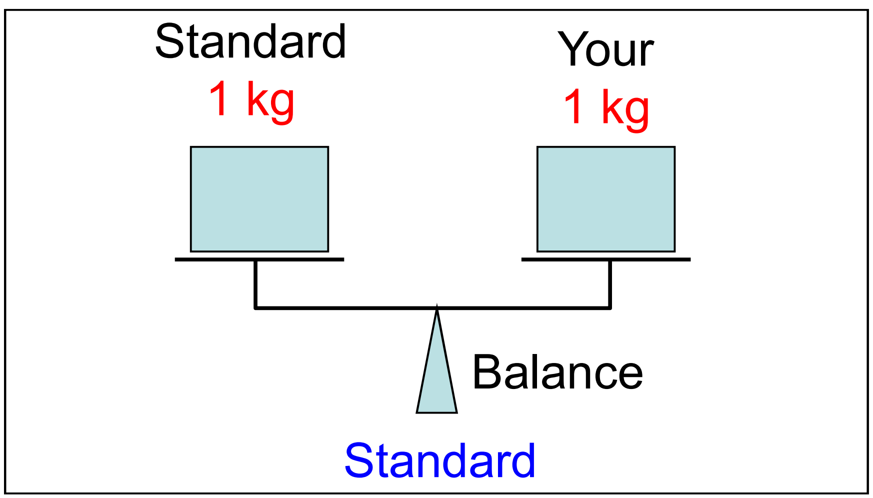
1-1 Measuring Things
Units



For the unit to be useful, people should agree on its definition.
We can use a standard or naturally occurring phenomenon to define a measuring unit.



A day is the time Earth takes to make one revolution about itself.

naturally occurring phenomenon

1-1 Measuring Things
Base/Fundemental physical quantities

There are many physical quantities, for example, pressure, mass, force, …

We can derive the units of these quantities from the units of a small number of physical quantities called base/fundemental physical quantities.

The selection of the base/fundemental physical quantities is not unique.

A set of base physical quantities has been selected by an agreement of the people.

In here, we will only deal with three base/fundemental physical quantities: length, mass, and time. The units of all other quantities can be derived from the units of these three quantities.

What is the unit of speed?

Derived physical quantity

Base physical quantities

$$\text{speed} = \frac{\text{length}}{\text{time}}$$

$$\text{unit of speed} = \frac{\text{unit of length}}{\text{unit of time}} = \frac{\text{meter}}{\text{second}}$$

1-2 The International System of Units

Seven base quantities

The International System of Units (the SI system of units) was established in 1971.

The SI system of units has seven base quantities.

In here, we will only deal with three base physical quantities: length, mass, and time. The units of all other quantities in here can be derived from the units of these three quantities.

Quantity	Unit name	Unit symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

For example, the SI unit of energy is the joule which can be written in terms of SI base units as follows

SI derived unit

SI base units

$$\text{joule} = \boxed{\frac{\text{kg m}^2}{\text{s}^2}}$$

One joule is one kilogram-meter squared per second squared.

1-2 The International System of Units
Meter, second, and kilogram

The meter is defined as the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.
The time interval was chosen so that the speed of light c is exactly
$c = 299\,792\,458$ m/s.

One second is defined as the time taken by 9 192 631 770 oscillations of the light emitted by the cesium atom.

The SI standard of mass is a platinum-iridium cylinder kept at the International Bureau of Weights and Measures near Paris and assigned a mass of 1 kilogram.

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, **the mass of carbon atom is used as a second mass standard**. By agreement,

mass of carbon atom = 12 atomic mass unit (u).

$$1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg}$$

1-2 The International System of Units

Scientific notation

Scientific notation is used to simplify expressing very large or very small quantities.

Quantity	In scientific notation	With prefixes
2 560 000 joule	2.56×10^{6} J	2.56 megajoule = 2.56 MJ
0.000 003 21 second	3.21×10^{-6} s	3.21 microsecond = 3.21 μs
5 460 meter	5.46×10^{3} m	5.46 kilometer = 5.46 km

The number is in scientific notation when it is expressed as some power of ten multiplied by another number between 1 and 10.

Factor	Prefix	Symbol
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{-2}	centi-	c
10^{-3}	mille-	m
10^{-6}	micro	μ
10^{-9}	nano-	n
10^{-12}	pico-	p

In some calculators, "exponent to ten" is written as "E"
6.52×10^{-7} written as 6.52 E-7

1-3 Changing Units

Conversion factor

$$3\text{ min} = (3\text{ min})(1) = (3\ \cancel{\text{min}})\left(\boxed{\frac{60\text{ s}}{1\ \cancel{\text{min}}}}\right) = 180\text{ s}$$

conversion factor

$$180\text{ s} = (180\text{ s})(1) = (180\ \cancel{\text{s}})\left(\boxed{\frac{1\text{ min}}{60\ \cancel{\text{s}}}}\right) = 3\text{ min}$$

A conversion factor is a ratio of units that is equal to one.

Multiplying any quantity by unity leaves the quantity unchanged.

Appendix D of your textbook gives conversion factors between SI and other system of units.

How many centimeters are there in 5.30 inches?

From Appendix D, 1 inch = 2.540 cm

$$5.30\text{ in} = (5.30\ \cancel{\text{in}})\left(\boxed{\frac{2.540\text{ cm}}{1\ \cancel{\text{in}}}}\right) = 13.5\text{ cm}$$

conversion factor

1-3 Changing Units

Example 1

A car moves at speed of 1.14 miles per minute. Use the following conversion factors to find its speed in kilometers per hour (km/h)

1 mile = 5280 feet

1 foot = 0.3048 meter

Solution

$$1.14\,\frac{\text{miles}}{\text{min}} = \left(1.14\,\frac{\cancel{\text{miles}}}{\cancel{\text{min}}}\right)\left(\frac{5280\ \cancel{\text{feet}}}{1\ \cancel{\text{mile}}}\right)\left(\frac{0.3048\ \cancel{\text{m}}}{1\ \cancel{\text{foot}}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right)$$

$$= 110\ \text{km/h}$$

Example 2

An aspirin tablet contains 325 mg of acetylsalicylic acid. Express this mass in grams.

Solution

Recall that prefix “milli” implies 10^{-3}, so m = 325 mg = 325 x 10^{-3} g = 0.325 g

1-3 Changing Units

Example 3

How many liters are there in one US fluid gallon, if

$1 \text{ US fluid gallon} = 231 \text{ in}^3$

$1 \text{ in} = 2.540 \text{ cm}$

$1 \text{ L} = 1000 \text{ cm}^3$?

Solution

$$1 \text{ gallon} = (1 \cancel{\text{gallon}})\left(\frac{231 \cancel{\text{in}^3}}{1 \cancel{\text{gallon}}}\right)\left(\frac{2.54 \cancel{\text{cm}}}{1 \cancel{\text{in}}}\right)^3\left(\frac{1 \text{ L}}{1000 \cancel{\text{cm}^3}}\right)$$

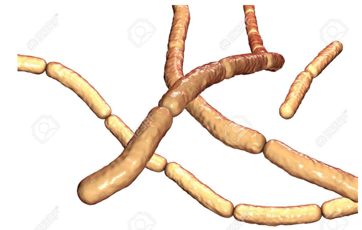
$$= 3.79 \text{ L}.$$

1-3 Changing Units

Example 4 Length of a bacteria

A bacteria has 50 nm length. If they are lined up

end to end, how many bacteria take place in 1 mm?



Solution

N = Number in 1 mm = ? L : Lenght of a bacteria

N x L = 1 mm

L = 50 nm = 50 x 10^{-9} m, length of a bacteria

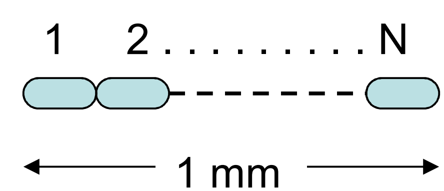
N x 50 nm = 1 mm

N = 1 mm /50 nm = 10^{-3} m /50 x 10^{-9} m

N = 0.02 x 10^{6} = 2 x 10^{4} = 20 000

Result : The number of bacteria in 1 mm is 20 000.

Recall
1 mm = 10^{-3} m
1 nm = 10^{-9} m



1-4 Dimensional Analysis
Dimensions

The dimension of a quantity is its property that we measure.
For distances, we measure length. → Dimension of distance = length
For periods, we measure time. → Dimension of period = time

Although any quantity might be measured in different units, it has one unique dimension. For example, a distance can be measured in meters or in feet. The dimension of distance is unique = length.

All quantities in here can be expressed in terms of three dimensions:

Length (L)
Time (T)
Mass (M)

The brackets [ ] is used to denote the dimension of a quantity.

[acceleration] stands for the dimension of acceleration

[distance] = Length = L

$$[\text{speed}] = \frac{\text{Length}}{\text{Time}} = \frac{L}{T}$$

[pure number] = 1
[angle] = 1
[argument of a trigonometric function] = 1

Quantities with dimension 1 are called dimensionless quantities.

1-4 Dimensional Analysis

Adding quantities

Quantities can be added or subtracted only if they have the same dimensions.

Acceptable

$$\boxed{x + v\,t}$$

$[x] = L$

$[v\,t] = \frac{L}{T}\,T = L$

The terms have the same dimensions.

Not acceptable

$$\cancel{\boxed{x + a\,t}}$$

$[x] = L$

$[a\,t] = \frac{L}{T^2}\,T = \frac{L}{T}$

The terms have different dimensions.

Given

x = distance	$[x] = L$
t = time	$[t] = T$
v = velocity	$[v] = \frac{L}{T}$
a = acceleration	$[a] = \frac{L}{T^2}$

1-4 Dimensional Analysis
Equating quantities

The terms on both sides of an equation must have the same dimensions.

Acceptable

$$\boxed{v = a\,t}$$

$$[v] = \frac{L}{T}$$

$$[a\,t] = \frac{L}{T^2}\,T = \frac{L}{T}$$

Both sides have the same dimensions.

Not acceptable

$$\cancel{\boxed{v = x\,t}}$$

$$[v] = \frac{L}{T}$$

$$[x\,t] = LT$$

The two sides have different dimensions.

Given

x = distance	$[x] = L$
t = time	$[t] = T$
v = velocity	$[v] = \frac{L}{T}$
a = acceleration	$[a] = \frac{L}{T^2}$

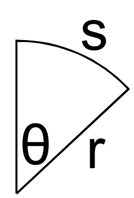
1-4 Dimensional Analysis
Equating quantities

The terms on both sides of an equation must have the same dimensions.

Not Acceptable

$$\boxed{\cancel{\text{area} = \theta\ r}}$$

$[\text{area}] = L^2$



$[\theta\ r] = 1\ L = L$

The two sides have different dimensions.

Not acceptable

$$\boxed{\cancel{\text{volume} = \pi\ r^2}}$$

$[\text{volume}] = L^3$

$[\pi\ r^2] = 1\ L^2 = L^2$

The two sides have different dimensions.

Given

r = radius $\quad [r] = L$

$[\text{area}] = [\text{distance}^2] = L^2$

$$\theta = \frac{s}{r}$$

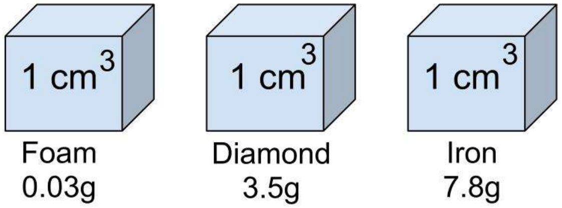
$$[\theta] = \frac{[\text{distance}]}{[\text{distance}]} = 1$$

$[\text{volume}] = [\text{distance}^3] = L^3$

1-4 Dimensional Analysis
Density

The density of a substance ρ (rho) is the amount of mass contained in a unit volume.
Mathematically, density is defined as mass divided by volume:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \qquad \rho = \frac{m}{V}$$



Q. What is the dimension analysis of density?

1-4 Dimensional Analysis
Example 5

Suppose the distance x is given in terms of acceleration a and time t as in the following expression

$$x = k\, a^{n}\ t^{m},$$

where k is a dimensionless constant. Find m and n.

Solution

Both sides of the equation should have the same dimensions.

$$[x] = L$$

$$[k\, a^{n}\ t^{m}] = (1)\left(\frac{L}{T^{2}}\right)^{n} T^{m} = L^{n}\ T^{m-2n}$$

$$L = L^{n}\ T^{m-2n}$$

$$L = L^{1}\ T^{0}$$

$$n = 1$$

$$m - 2n = 0 \rightarrow m = 2n = 2$$

$$x = k\, a\, t^{2}$$

1-4 Dimensional Analysis

Example 6

Suppose the acceleration a of a particle moving with uniform speed v in a circle of radius r is given by

$$a = k\, v^n\, r^m,$$

where k is a dimensionless constant. Find m and n.

Solution

Both sides of the equation should have the same dimensions.

$$[a] = \frac{L}{T^2}$$

$$[k\, v^n\, r^m] = (1)\left(\frac{L}{T}\right)^n L^m = \frac{L^{n+m}}{T^n}$$

$$\frac{L}{T^2} = \frac{L^{n+m}}{T^n}$$

$$n = 2$$

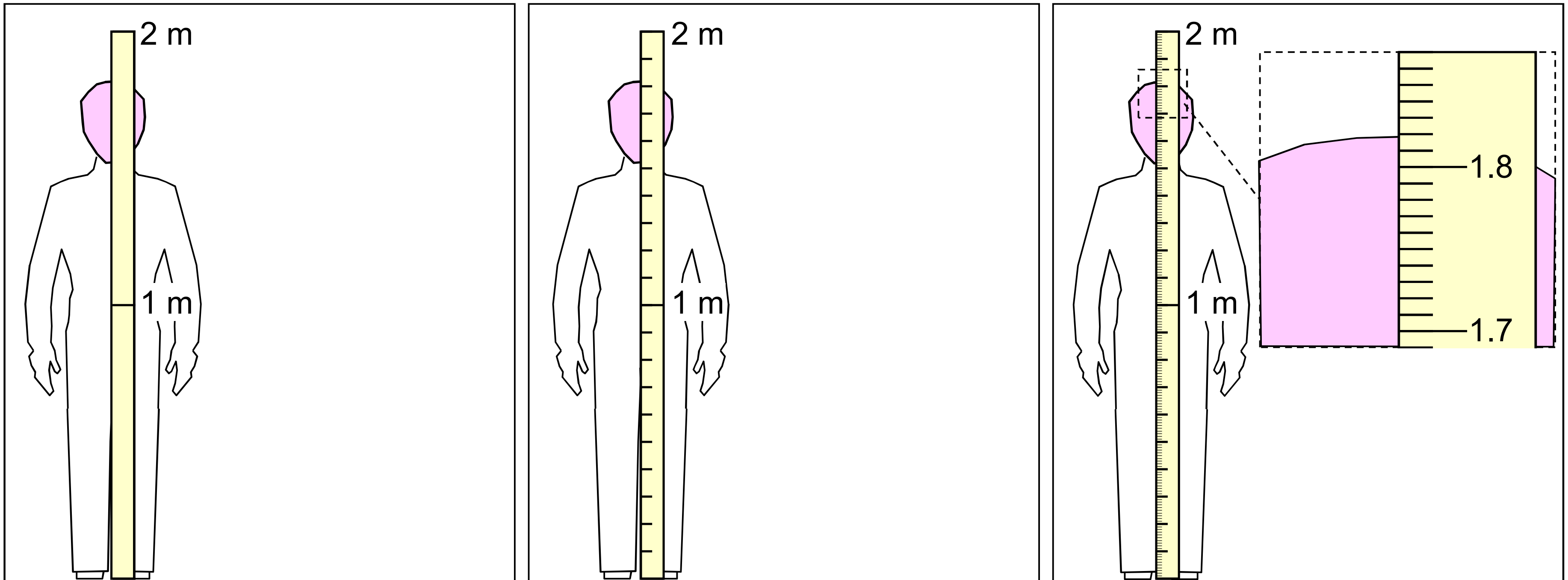
$$n + m = 1 \quad \rightarrow \quad m = 1 - n = -1$$

$$a = k\,\frac{v^2}{r}$$

1-5 Significant Figures
Measurements

Numbers

Exact	Inexact
Five fingers in your hand	Any measurement



Height = 1.8 m
Correct digit: 1
doubtful digit: 8
2 significant figures

Height = 1.82 m
Correct digits: 1.8
doubtful digit: 2
3 significant figures

Height = 1.819 m
Correct digits: 1.81
doubtful digit: 9
4 significant figures

Number of significant figures depends on the instrument used in the measurement.

1-5 Significant Figures
Number of significant figures

A significant figure is a digit in a number.

15.07 This number has four significant figures.

The least significant figure is the significant figure farthest to the right.

10.68 The 8 is the least significant figure.

All leading zeros are not significant figures.

0.00064 This number has two significant figures.

All trailing zeros to the right of the decimal point are significant figures

12.000 This number has five significant figures.

3000 The trailing zeros to the left of the decimal point might or might not be significant figures.
The zeros might not be significant and they are just being used to locate the decimal point.
However, in this course, we will take them as significant figures.

1-5 Significant Figures
Rounding off

When the left-most of the digits to be discarded is 5 or more, the last remaining digit is rounded up; otherwise it is retained as is.

Round up

$$2.36502 = 2.37$$

5 or more

Do not change

$$80.76493 = 80.76$$

Less than 5

1-5 Significant Figures
Multiplication or division

When multiplying or dividing quantities, the result should have the same number of significant figures as the quantity with the lowest number of significant figures.

$2.31563 \times 0.25 = 0.58$

6 significant figures

2 significant figures

Your calculator gives 0.578908.

You should round off your answer to two significant figures.

$2.751 \div 3.22 = 0.854$

4 significant figures

3 significant figures

Your calculator gives 0.854348.

You should round off your answer to three significant figures.

1-5 Significant Figures

Addition or subtraction

When adding or subtracting quantities, the least significant figure in the result has the same position relative to the decimal point as that of the quantity whose least significant figure is farthest to the left.

$$\begin{array}{r} 9.1 \\ +\ 7.265 \\ \hline 16.4 \end{array}$$

Least significant figure (9.1)
Least significant figure (7.265)
Least significant figure (16.4)

Your calculator gives 16.365.

Since 9.1 is the quantity with its least significant figure farthest to the left relative to the decimal point, your answer should be rounded off so that the position of its least significant figure match that of 9.1.

$$\begin{array}{r} 1.02 \\ -\ 0.9743 \\ \hline 0.05 \end{array}$$

Least significant figure (1.02)
Least significant figure (0.9743)
Least significant figure (0.05)

Your calculator gives 0.0457.

Since 1.02 is the quantity with its least significant figure farthest to the left relative to the decimal point, your answer should be rounded off so that the position of its least significant figure match that of 1.02.

1-6 Order-of-Magnitude Calculations

Order-of-magnitude

An order-of magnitude calculation is a rough estimate that is accurate to within a factor of about 10.
It is useful if you want to get a quick rough answer.
You may use this estimate to check your detailed calculation.

The order of magnitude of a quantity is the power of ten when the quantity is expressed in scientific notation.

$A = 7\ 600 = 7.6 \times 10^3$	The order of magnitude of A is 3
$B = 3\ 700 = 3.7 \times 10^3$	The order of magnitude of B is 3

$A = 7\ 600 \approx 10\ 000 = 10^4$	The nearest order of magnitude of A is 4
$B = 3\ 600 \approx 1\ 000 = 10^3$	The nearest order of magnitude of B is 3

1-6 Order-of-Magnitude Calculations

Example 7

Estimate the number of heart beats during an average human lifetime.

Solution

Average human lifetime ≈ 70 years

Average heart beat per minute ≈ 70 beats

Number of days per year = 365 days ≈ 400 days .

Number of hours per day = 24 hours ≈ 20 hours .

Number of minutes per hour = 60 minutes

Number of heart beats during human lifetime ≈

$$\left(70\,\frac{\text{beats}}{\cancel{\text{min}}}\right)\left(60\,\frac{\cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(20\,\frac{\cancel{\text{h}}}{\cancel{\text{day}}}\right)\left(400\,\frac{\cancel{\text{day}}}{\cancel{\text{yr}}}\right)\left(70\,\frac{\cancel{\text{yr}}}{\text{liftime}}\right) = \boxed{2\times10^{9}}\,\frac{\text{beats}}{\text{lifetime}}\,.$$

Reasonable!

Compare this estimate with the detailed calculation

$$\left(70\,\frac{\text{beats}}{\text{min}}\right)\left(60\,\frac{\text{min}}{\text{h}}\right)\left(24\,\frac{\text{h}}{\text{day}}\right)\left(365\,\frac{\text{day}}{\text{yr}}\right)\left(70\,\frac{\text{yr}}{\text{liftime}}\right) = \boxed{2.5\times10^{9}}\,\frac{\text{beats}}{\text{lifetime}}\,.$$