

Chapter 2 Motion and Recombination of Electrons and Holes

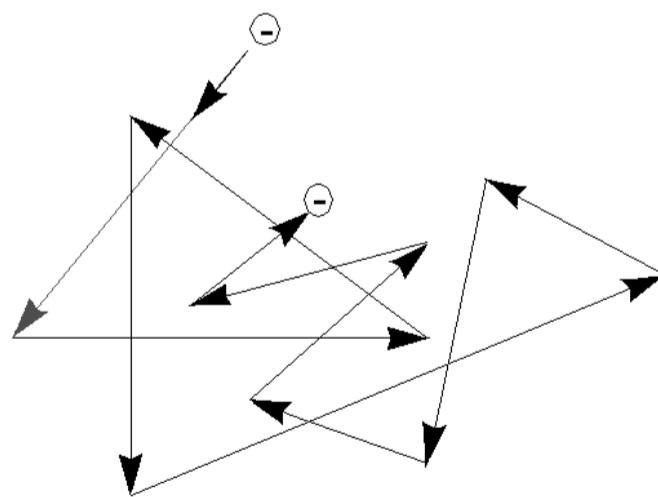
2.1 Thermal Motion

$$\text{Average electron or hole kinetic energy} = \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$$

$$v_{th} = \sqrt{\frac{3kT}{m_n}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ JK}^{-1} \times 300 \text{ K}}{0.26 \times 9.1 \times 10^{-31} \text{ kg}}}$$
$$= 2.3 \times 10^5 \text{ m/s} = 2.3 \times 10^7 \text{ cm/s}$$

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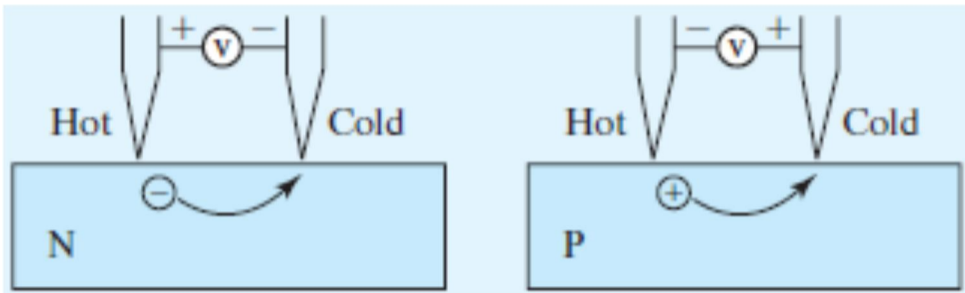
2.1 Thermal Motion



- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero.
- Mean time between collisions is $\tau_m \sim 0.1 \text{ ps}$

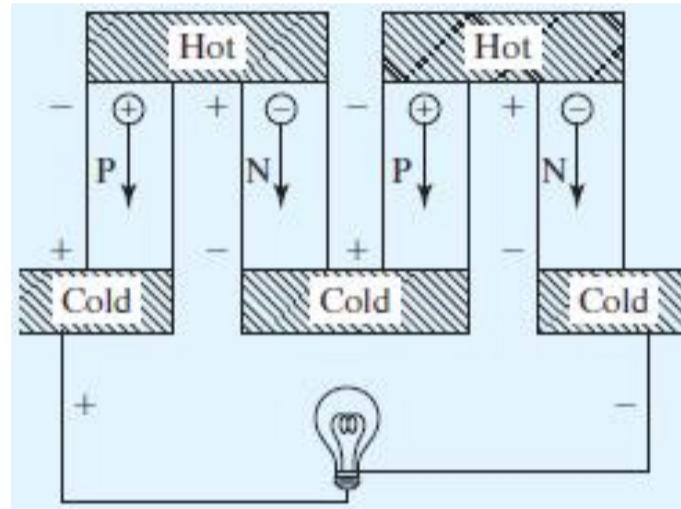
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Hot-point Probe can determine sample doing type



Hot-point Probe distinguishes N and P type semiconductors.

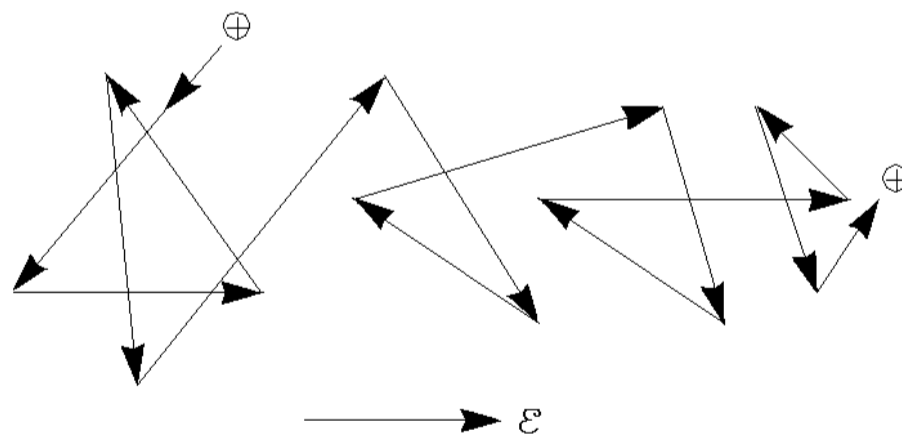
Thermoelectric Generator (from heat to electricity) and Cooler (from electricity to refrigeration)



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2.2 Drift

2.2.1 Electron and Hole Mobilities



- *Drift* is the motion caused by an electric field.

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2.2.1 Electron and Hole Mobilities

$$m_p v = q E \tau_{mp}$$

$$v = \frac{q E \tau_{mp}}{m_p}$$

$$v = \mu_p E$$

$$\mu_p = \frac{q \tau_{mp}}{m_p}$$

$$v = -\mu_n E$$

$$\mu_n = \frac{q \tau_{mn}}{m_n}$$

- μ_p is the hole mobility and μ_n is the electron mobility

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2.2.1 Electron and Hole Mobilities

$$v = \mu E ; \mu \text{ has the dimensions of } v/E \left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right].$$

Electron and hole mobilities of selected semiconductors

	Si	Ge	GaAs	InAs
μ_n (cm ² /V·s)	1400	3900	8500	30000
μ_p (cm ² /V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

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Drift Velocity, Mean Free Time, Mean Free Path

EXAMPLE: Given $\mu_p = 470 \text{ cm}^2/\text{V}\cdot\text{s}$, what is the hole drift velocity at $E = 10^3 \text{ V/cm}$? What is τ_{mp} and what is the distance traveled between collisions (called the **mean free path**)? Hint: When in doubt, use the MKS system of units.

Solution: $v = \mu_p E = 470 \text{ cm}^2/\text{V}\cdot\text{s} \times 10^3 \text{ V/cm} = 4.7 \times 10^5 \text{ cm/s}$

$$\begin{aligned} \tau_{mp} &= \mu_p m_p / q = 470 \text{ cm}^2/\text{V}\cdot\text{s} \times 0.39 \times 9.1 \times 10^{-31} \text{ kg} / 1.6 \times 10^{-19} \text{ C} \\ &= 0.047 \text{ m}^2/\text{V}\cdot\text{s} \times 2.2 \times 10^{-12} \text{ kg/C} = 1 \times 10^{-13} \text{ s} = 0.1 \text{ ps} \end{aligned}$$

$$\begin{aligned} \text{mean free path} &= \tau_{mh} v_{th} \sim 1 \times 10^{-13} \text{ s} \times 2.2 \times 10^7 \text{ cm/s} \\ &= 2.2 \times 10^{-6} \text{ cm} = 220 \text{ \AA} = 22 \text{ nm} \end{aligned}$$

This is smaller than the typical dimensions of devices, but getting close.

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2.2.2 Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:

1. Phonon Scattering
2. Ionized-Impurity (Coulombic) Scattering

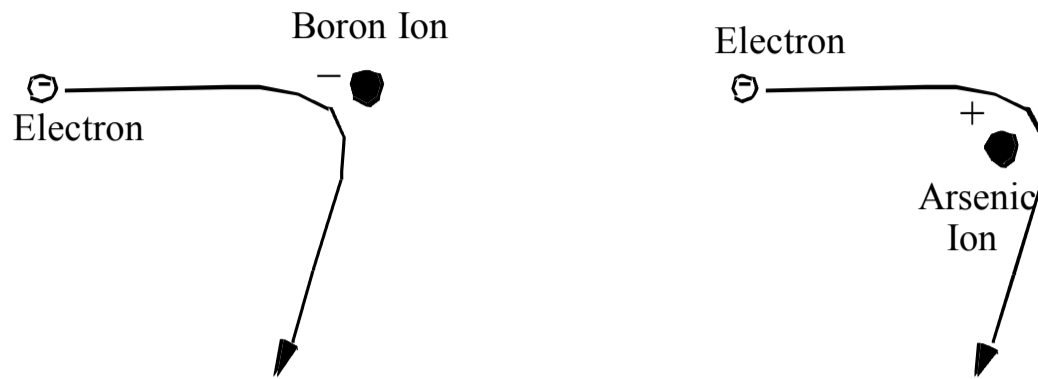
Phonon scattering mobility decreases when temperature rises:

$$\mu_{\text{phonon}} \propto \tau_{\text{phonon}} \propto \frac{1}{\text{phonon density} \times \text{carrier thermal velocity}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$\mu = q\tau/m$ $\propto T$ $v_{th} \propto T^{1/2}$

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Impurity (Dopant)-Ion Scattering or Coulombic Scattering

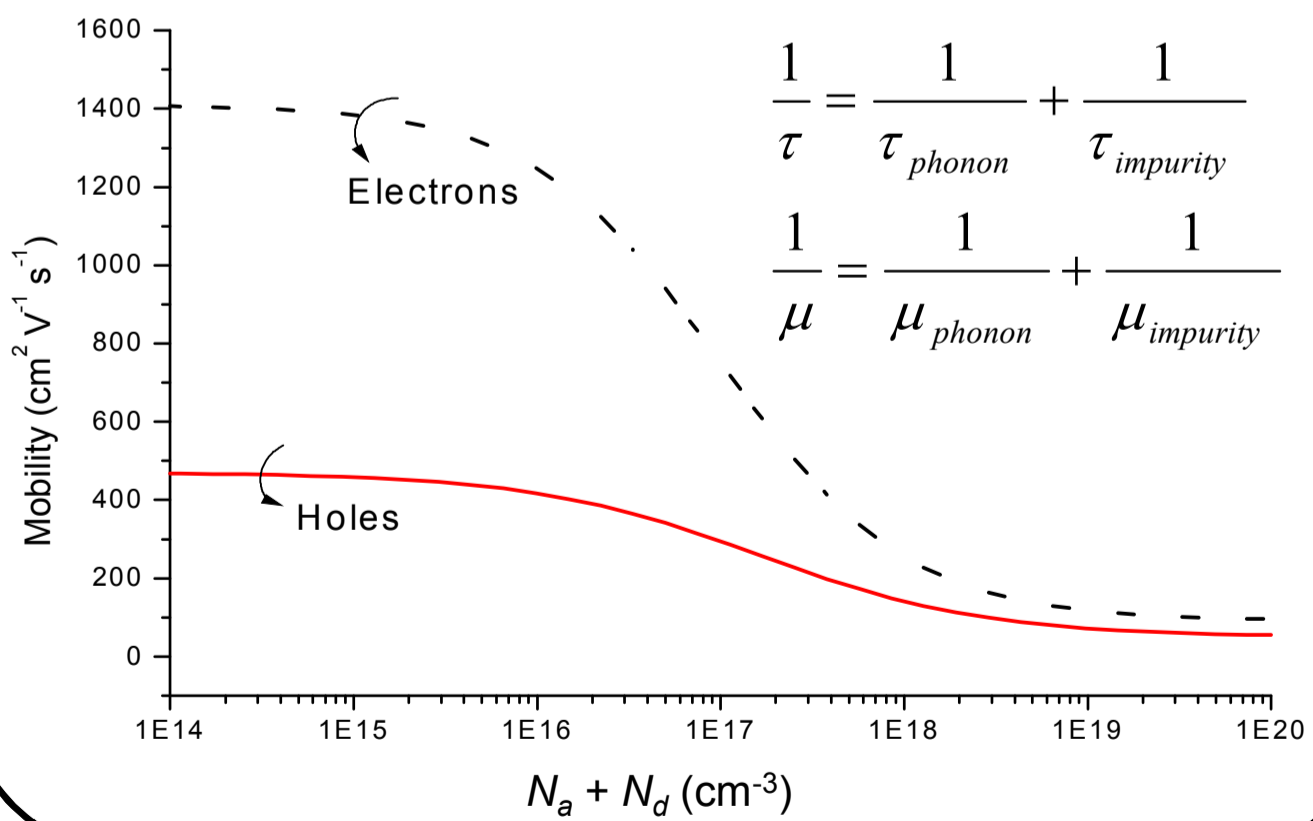


There is less change in the direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

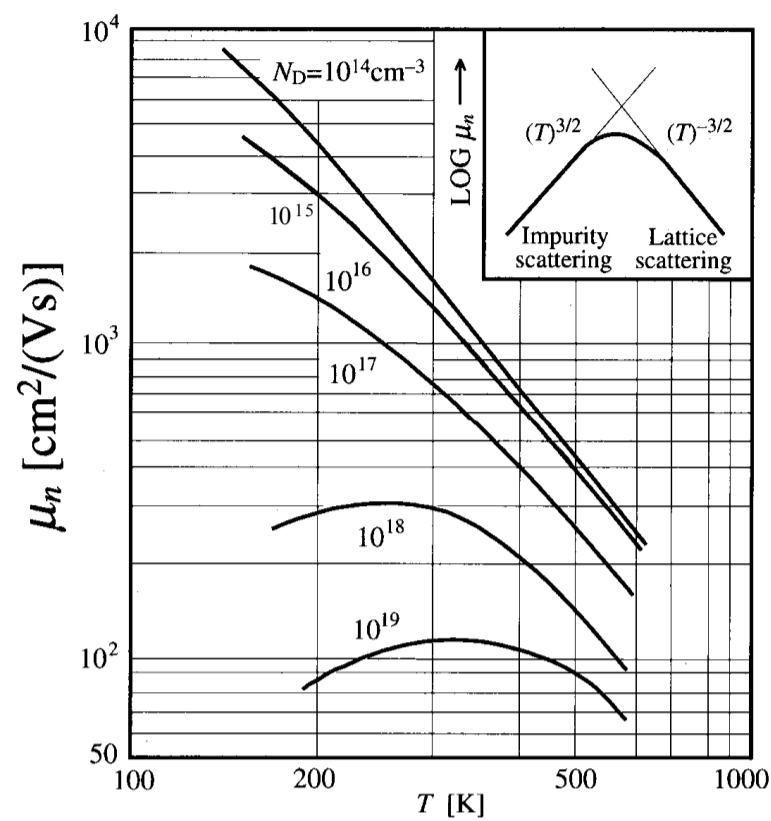
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Total Mobility



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Temperature Effect on Mobility



Question:

What N_d will make $d\mu_n/dT = 0$ at room temperature?

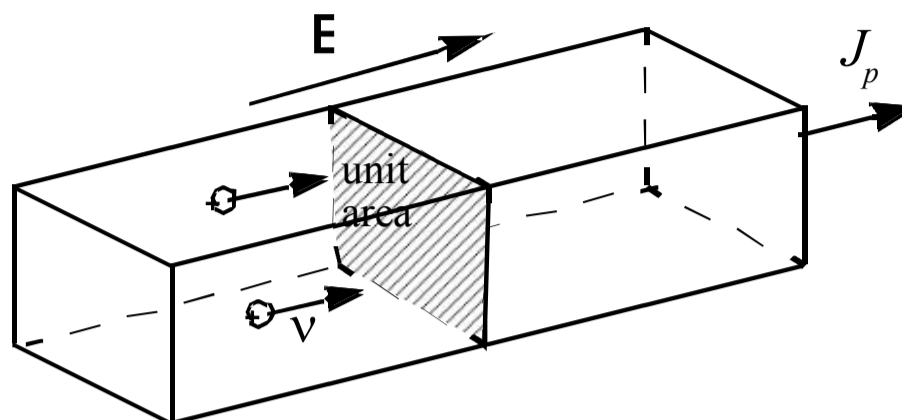
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Velocity Saturation

- When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy.
- Therefore, the kinetic energy is capped at large \mathbf{E} , and the velocity does not rise above a saturation velocity, v_{sat} .
- **Velocity saturation** has a deleterious effect on device speed as shown in Ch. 6.

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2.2.3 Drift Current and Conductivity



Hole current density $J_p = qp v$ A/cm² or C/cm²·sec

EXAMPLE: If $p = 10^{15} \text{cm}^{-3}$ and $v = 10^4 \text{cm/s}$, then
 $J_p = 1.6 \times 10^{-19} \text{C} \times 10^{15} \text{cm}^{-3} \times 10^4 \text{cm/s}$
 $= 1.6 \text{C/s} \cdot \text{cm}^2 = 1.6 \text{A/cm}^2$

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2.2.3 Drift Current and Conductivity

$$J_{p,drift} = qp v = qp \mu_p \mathbf{E}$$

$$J_{n,drift} = -qn v = qn \mu_n \mathbf{E}$$

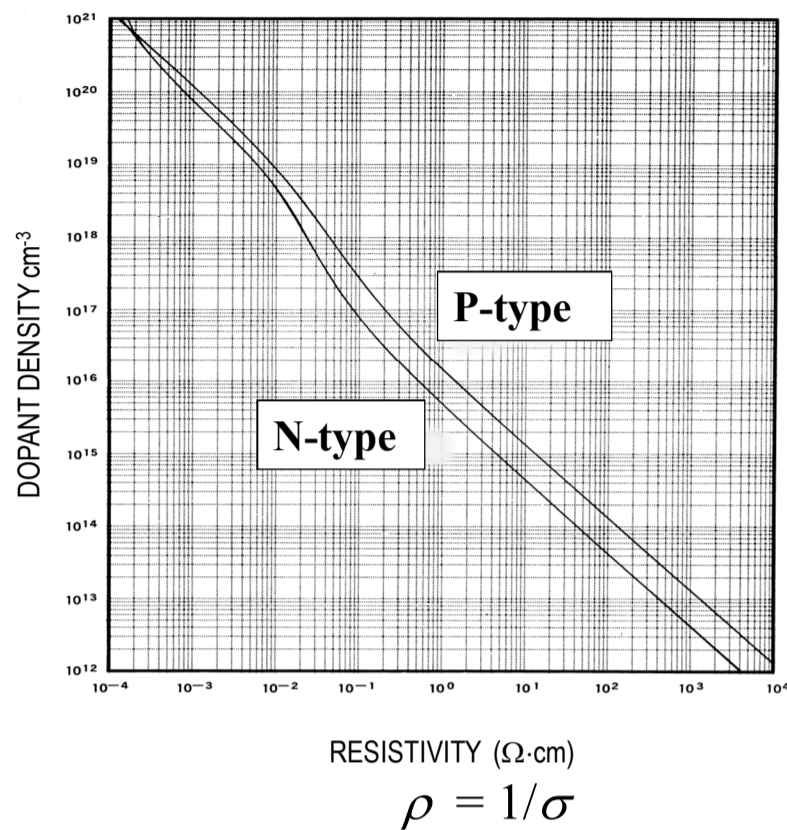
$$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma \mathbf{E} = (qn \mu_n + qp \mu_p) \mathbf{E}$$

\therefore **conductivity** (1/ohm-cm) of a semiconductor is
 $\sigma = qn \mu_n + qp \mu_p$

$1/\sigma =$ is resistivity (ohm-cm)

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Relationship between Resistivity and Dopant Density



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EXAMPLE: Temperature Dependence of Resistance

(a) What is the resistivity (ρ) of silicon doped with 10^{17}cm^{-3} of arsenic?

(b) What is the resistance (R) of a piece of this silicon material $1\mu\text{m}$ long and $0.1\mu\text{m}^2$ in cross-sectional area?

Solution:

(a) Using the N-type curve in the previous figure, we find that $\rho = 0.084\ \Omega\cdot\text{cm}$.

$$\begin{aligned} (b) R &= \rho L/A = 0.084\ \Omega\cdot\text{cm} \times 1\ \mu\text{m} / 0.1\ \mu\text{m}^2 \\ &= 0.084\ \Omega\cdot\text{cm} \times 10^{-4}\ \text{cm} / 10^{-10}\ \text{cm}^2 \\ &= 8.4 \times 10^{-4}\ \Omega \end{aligned}$$

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EXAMPLE: Temperature Dependence of Resistance

By what factor will R increase or decrease from $T=300\text{ K}$ to $T=400\text{ K}$?

Solution: *The temperature dependent factor in σ (and therefore ρ) is μ_n . From the mobility vs. temperature curve for 10^{17} cm^{-3} , we find that μ_n decreases from 770 at 300K to 400 at 400K. As a result, R **increases** by*

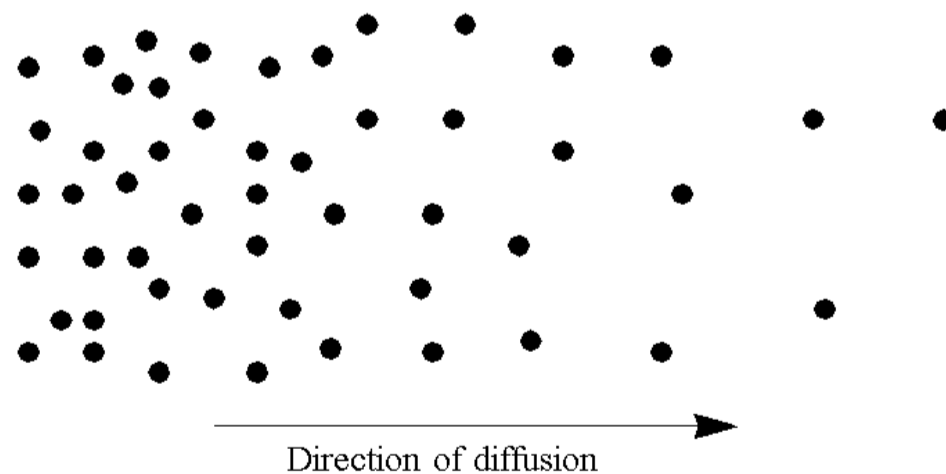
$$\frac{770}{400} = 1.93$$

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2.3 Diffusion Current

Higher particle concentration

Lower particle concentration



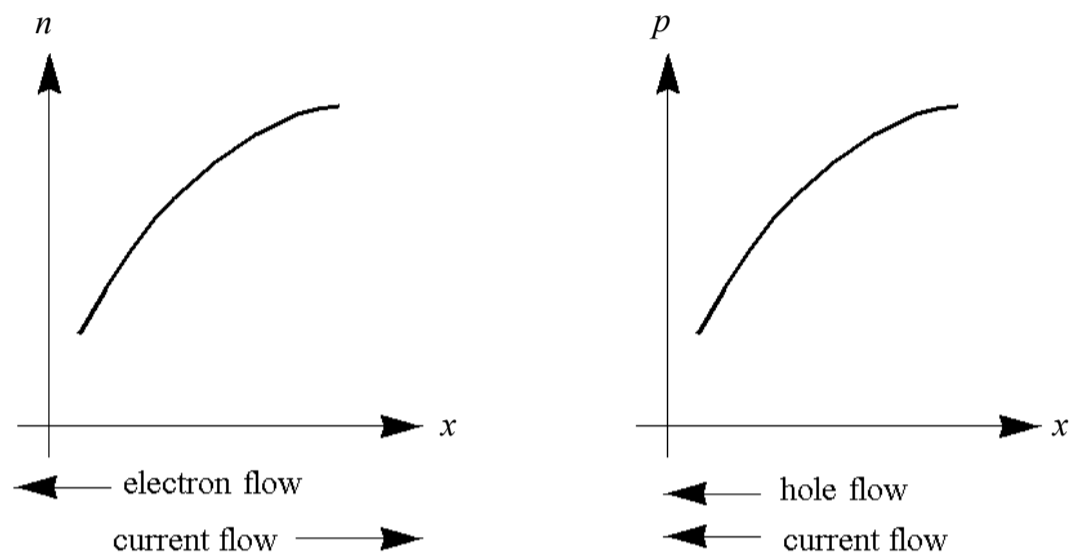
Particles diffuse from a higher-concentration location to a lower-concentration location.

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2.3 Diffusion Current

$$J_{n,diffusion} = qD_n \frac{dn}{dx} \qquad J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

D is called the diffusion constant. Signs explained:



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Total Current – Review of Four Current Components

$$J_{TOTAL} = J_n + J_p$$

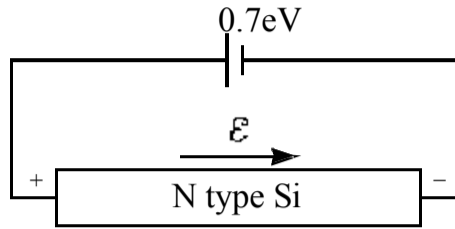
$$J_n = J_{n,drift} + J_{n,diffusion} = qn\mu_n \mathbf{E} + qD_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diffusion} = qp\mu_p \mathbf{E} - qD_p \frac{dp}{dx}$$

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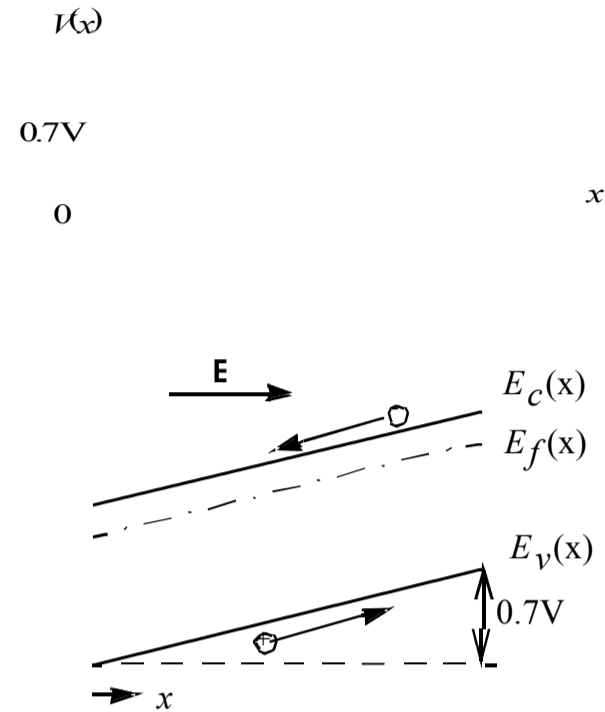
2.4 Relation Between the Energy

Diagram and V, E



E_c and E_v vary in the opposite direction from the voltage. That is, E_c and E_v are higher where the voltage is lower.

$$\mathbf{E}(x) = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx}$$



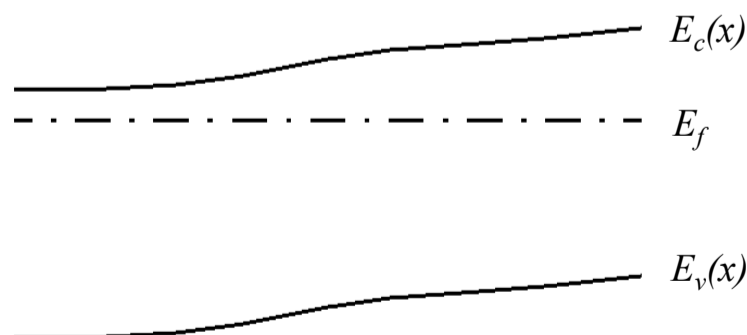
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2.5 Einstein Relationship between D and μ

Consider a piece of non-uniformly doped semiconductor.

N-type semiconductor

Decreasing donor concentration



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q \mathbf{E}$$

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2.5 Einstein Relationship between D and μ

$$\frac{dn}{dx} = -\frac{n}{kT} qE$$

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx} = 0 \quad \text{at equilibrium.}$$

$$0 = qn\mu_n E - qn \frac{qD_n}{kT} E$$

$$D_n = \frac{kT}{q} \mu_n$$

Similarly,

$$D_p = \frac{kT}{q} \mu_p$$

*These are known as the **Einstein relationship**.*

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EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with $\mu_p = 410 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$?

Solution:

$$D_p = \left(\frac{kT}{q} \right) \mu_p = (26 \text{ mV}) \cdot 410 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} = 11 \text{ cm}^2/\text{s}$$

Remember: $kT/q = 26 \text{ mV}$ at room temperature.

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2.6 *Electron-Hole Recombination*

- The equilibrium carrier concentrations are denoted with n_0 and p_0 .
- The total electron and hole concentrations can be different from n_0 and p_0 .
- The differences are called the *excess carrier concentrations* n' and p' .

$$\begin{aligned} n &\equiv n_0 + n' \\ p &\equiv p_0 + p' \end{aligned}$$

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Charge Neutrality

- Charge neutrality is satisfied at equilibrium ($n' = p' = 0$).
- When a non-zero n' is present, an equal p' may be assumed to be present to maintain charge equality and vice-versa.
- If charge neutrality is not satisfied, the net charge will attract or repel the (majority) carriers through the drift current until neutrality is restored.

$$n' = p'$$

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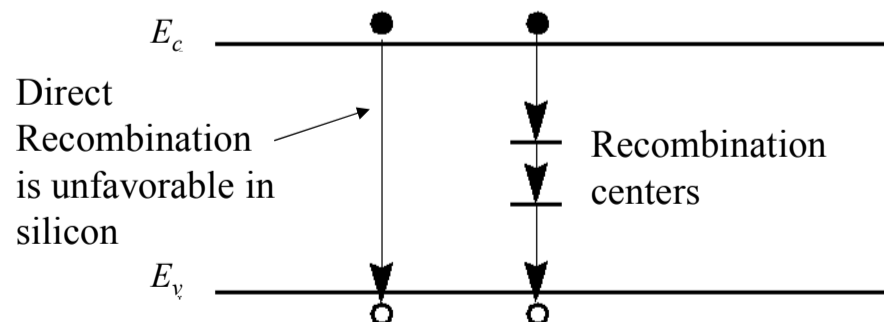
Recombination Lifetime

- Assume light generates n' and p' . If the light is suddenly turned off, n' and p' decay with time until they become zero.
- The process of decay is called *recombination*.
- The time constant of decay is the *recombination time* or *carrier lifetime*, τ .
- Recombination is nature's way of restoring equilibrium ($n' = p' = 0$).

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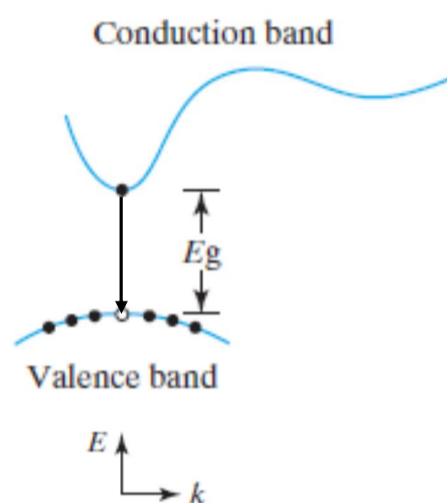
Recombination Lifetime

- τ ranges from 1ns to 1ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt.
- These *deep traps* capture electrons and holes to facilitate recombination and are called *recombination centers*.



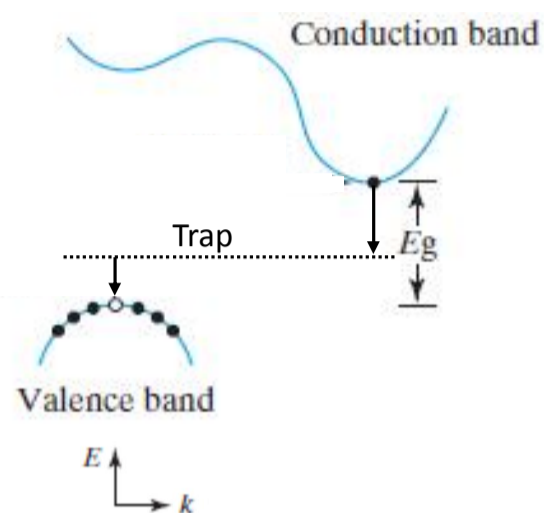
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Direct and Indirect Band Gap



Direct band gap
Example: GaAs

Direct recombination is efficient as k conservation is satisfied.



Indirect band gap
Example: Si

Direct recombination is rare as k conservation is not satisfied

Rate of recombination ($s^{-1}cm^{-3}$)

$$\frac{dn'}{dt} = -\frac{n'}{\tau}$$

$$n' = p'$$

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} = \frac{dp'}{dt}$$

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EXAMPLE: Photoconductors

A bar of Si is doped with boron at 10^{15}cm^{-3} . It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of $10^{20}/\text{s}\cdot\text{cm}^3$. The recombination lifetime is $10\mu\text{s}$. What are (a) p_0 , (b) n_0 , (c) p' , (d) n' , (e) p , (f) n , and (g) the np product?

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EXAMPLE: Photoconductors

Solution:

(a) What is p_0 ?

$$p_0 = N_a = 10^{15} \text{ cm}^{-3}$$

(b) What is n_0 ?

$$n_0 = n_i^2/p_0 = 10^5 \text{ cm}^{-3}$$

(c) What is p' ?

In steady-state, the rate of generation is equal to the rate of recombination.

$$\begin{aligned} 10^{20}/\text{s}\cdot\text{cm}^3 &= p'/\tau \\ \therefore p' &= 10^{20}/\text{s}\cdot\text{cm}^3 \cdot 10^{-5}\text{s} = 10^{15} \text{ cm}^{-3} \end{aligned}$$

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EXAMPLE: Photoconductors

(d) *What is n' ?*

$$n' = p' = 10^{15} \text{ cm}^{-3}$$

(e) *What is p ?*

$$p = p_0 + p' = 10^{15} \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} = 2 \times 10^{15} \text{ cm}^{-3}$$

(f) *What is n ?*

$$n = n_0 + n' = 10^5 \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} \sim 10^{15} \text{ cm}^{-3} \text{ since } n_0 \ll n'$$

(g) *What is np ?*

$$np \sim 2 \times 10^{15} \text{ cm}^{-3} \cdot 10^{15} \text{ cm}^{-3} = 2 \times 10^{30} \text{ cm}^{-6} \gg n_i^2 = 10^{20} \text{ cm}^{-6}.$$

The np product can be very different from n_i^2 .

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2.7 Thermal Generation

If n' is negative, there are fewer electrons than the equilibrium value.

As a result, there is a net rate of ***thermal generation*** at the rate of $|n'|/\tau$.

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2.8 Quasi-equilibrium and Quasi-Fermi Levels

- Whenever $n' = p' \neq 0$, $np \neq n_i^2$. We would like to preserve and use the simple relations:

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

- But these equations lead to $np = n_i^2$. The solution is to introduce two **quasi-Fermi levels** E_{fn} and E_{fp} such that

$$n = N_c e^{-(E_c - E_{fn})/kT}$$

$$p = N_v e^{-(E_{fp} - E_v)/kT}$$

Even when electrons and holes are not at equilibrium, *within each group* the carriers can be at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.

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EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Consider a Si sample with $N_d = 10^{17} \text{ cm}^{-3}$ and $n' = p' = 10^{15} \text{ cm}^{-3}$.

(a) Find E_f .

$$n = N_d = 10^{17} \text{ cm}^{-3} = N_c \exp[-(E_c - E_f)/kT]$$

$$\therefore E_c - E_f = 0.15 \text{ eV. } (E_f \text{ is below } E_c \text{ by } 0.15 \text{ eV.})$$

Note: n' and p' are much less than the majority carrier concentration. This condition is called **low-level injection**.

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EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Now assume $n' = p' = 10^{15} \text{ cm}^{-3}$.

(b) Find E_{fn} and E_{fp} .

$$n = 1.01 \times 10^{17} \text{ cm}^{-3} = N_c e^{-(E_c - E_{fn})/kT}$$

$$\begin{aligned} \therefore E_c - E_{fn} &= kT \times \ln(N_c / 1.01 \times 10^{17} \text{ cm}^{-3}) \\ &= 26 \text{ meV} \times \ln(2.8 \times 10^{19} \text{ cm}^{-3} / 1.01 \times 10^{17} \text{ cm}^{-3}) \\ &= 0.15 \text{ eV} \end{aligned}$$

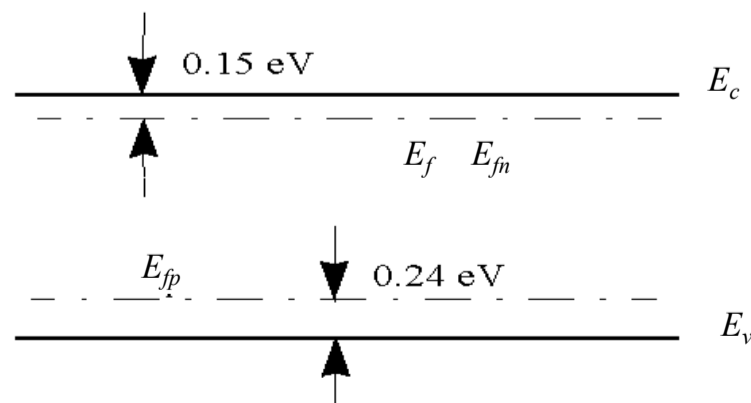
E_{fn} is nearly identical to E_f because $n \approx n_0$.

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EXAMPLE: Quasi-Fermi Levels

$$p = 10^{15} \text{ cm}^{-3} = N_v e^{-(E_{fp} - E_v)/kT}$$

$$\begin{aligned} \therefore E_{fp} - E_v &= kT \times \ln(N_v / 10^{15} \text{ cm}^{-3}) \\ &= 26 \text{ meV} \times \ln(1.04 \times 10^{19} \text{ cm}^{-3} / 10^{15} \text{ cm}^{-3}) \\ &= 0.24 \text{ eV} \end{aligned}$$



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2.9 Chapter Summary

$$v_p = \mu_p \mathbf{E}$$

$$v_n = -\mu_n \mathbf{E}$$

$$J_{p,drift} = qp\mu_p \mathbf{E}$$

$$J_{n,drift} = qn\mu_n \mathbf{E}$$

$$J_{n,diffusion} = qD_n \frac{dn}{dx}$$

$$J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

$$D_n = \frac{kT}{q} \mu_n$$

$$D_p = \frac{kT}{q} \mu_p$$

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2.9 Chapter Summary

τ is the recombination lifetime.

n' and p' are the *excess carrier concentrations*.

$$n = n_0 + n'$$

$$p = p_0 + p'$$

Charge neutrality requires $n' = p'$.

$$\text{rate of recombination} = n'/\tau = p'/\tau$$

E_{fn} and E_{fp} are the quasi-Fermi levels of electrons and holes.

$$n = N_c e^{-(E_c - E_{fn})/kT}$$

$$p = N_v e^{-(E_{fp} - E_v)/kT}$$

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