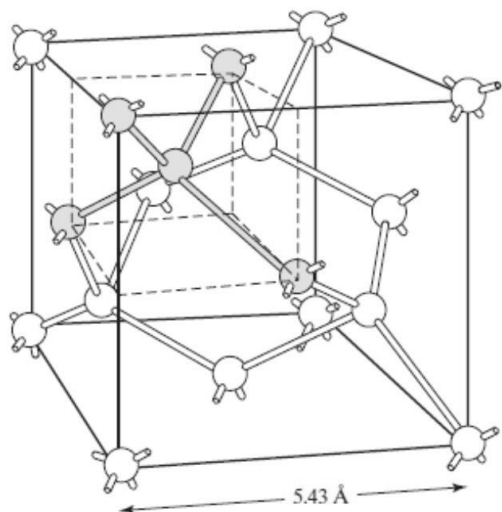


Chapter 1 *Electrons and Holes in Semiconductors*

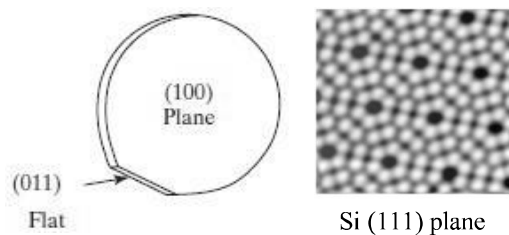
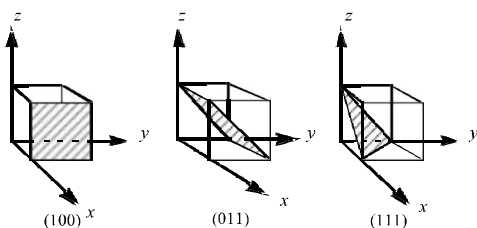
1.1 *Silicon Crystal Structure*

- *Unit cell* of silicon crystal is cubic.
- *Each Si atom has 4 nearest neighbors.*



Slide 1-1

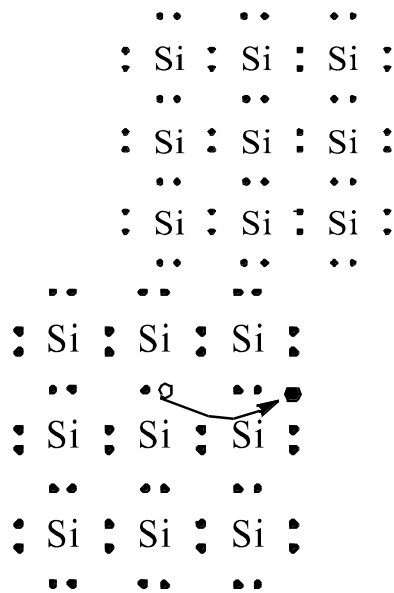
Silicon Wafers and Crystal Planes



- The standard notation for crystal planes is based on the cubic unit cell.
- Silicon wafers are usually cut along the (100) plane with a flat or notch to help orient the wafer during IC fabrication.

Slide 1-2

1.2 Bond Model of Electrons and Holes

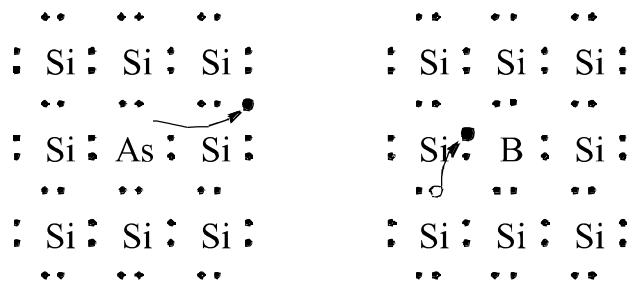


- Silicon crystal in a two-dimensional representation.

- When an electron breaks loose and becomes a *conduction electron*, a *hole* is also created.

Slide 1-3

Dopants in Silicon

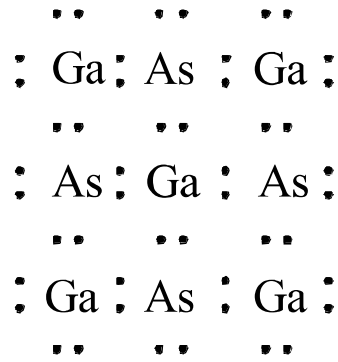
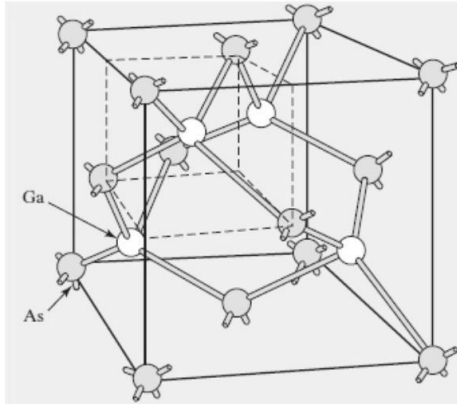


- As, a Group V element, introduces conduction electrons and creates *N-type silicon*, and is called a *donor*.
- B, a Group III element, introduces holes and creates *P-type silicon*, and is called an *acceptor*.
- Donors and acceptors are known as dopants. Dopant ionization energy $\sim 50\text{meV}$ (very low).

Hydrogen:
$$E_{ion} = \frac{m_0 q^4}{8\epsilon_0^2 h^2} = 13.6 \text{ eV}$$

Slide 1-4

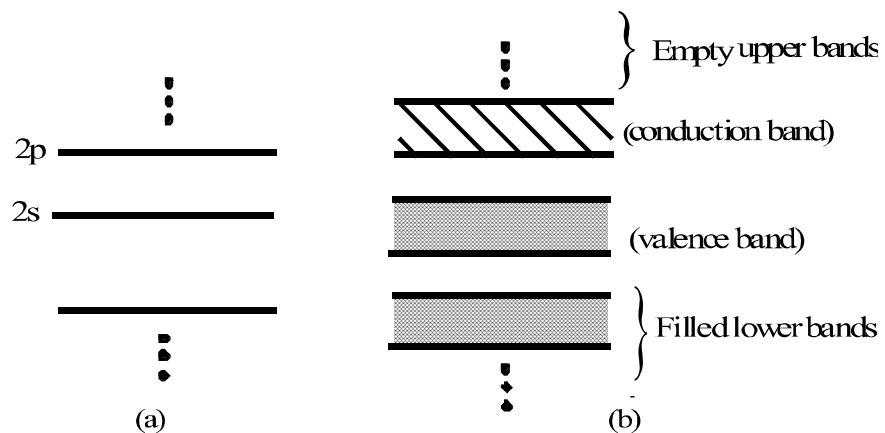
GaAs, III-V Compound Semiconductors, and Their Dopants



- GaAs has the same crystal structure as Si.
- GaAs, GaP, GaN are III-V compound semiconductors, important for optoelectronics.
- Which group of elements are candidates for donors? acceptors?

Slide 1-5

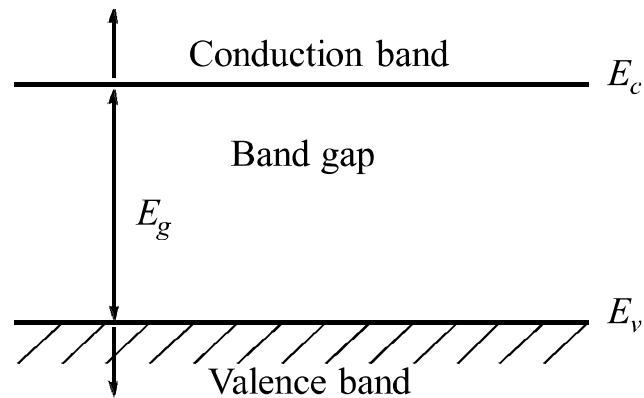
1.3 Energy Band Model



- Energy states of Si atom (a) expand into energy bands of Si crystal (b).
- The lower bands are filled and higher bands are empty in a semiconductor.
- The highest filled band is the **valence band**.
- The lowest empty band is the **conduction band**.

Slide 1-6

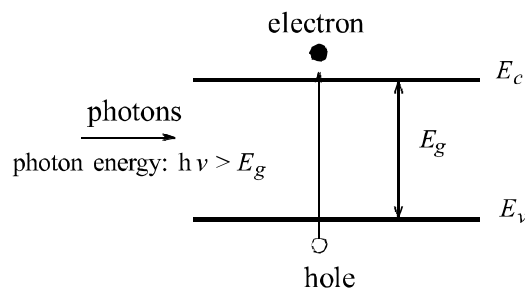
1.3.1 Energy Band Diagram



- **Energy band diagram** shows the bottom edge of conduction band, E_c , and top edge of valence band, E_v .
- E_c and E_v are separated by the **band gap energy**, E_g .

Slide 1-7

Measuring the Band Gap Energy by Light Absorption



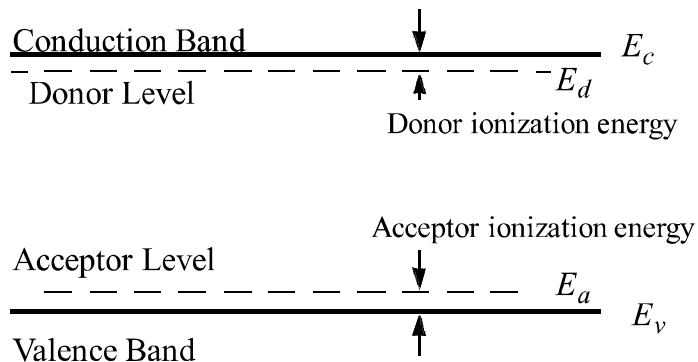
- E_g can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

Bandgap energies of selected semiconductors

Semi-conductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
E_g (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6

Slide 1-8

1.3.2 Donor and Acceptor in the Band Model

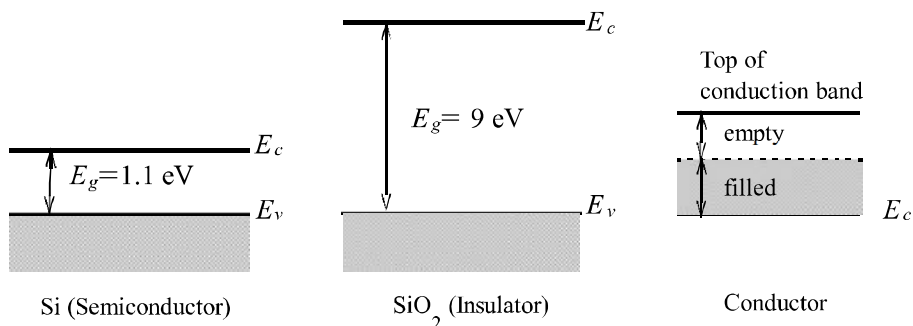


Ionization energy of selected donors and acceptors in silicon

Dopant	Donors			Acceptors		
	Sb	P	As	B	Al	In
Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV)	39	44	54	45	57	160

Slide 1-9

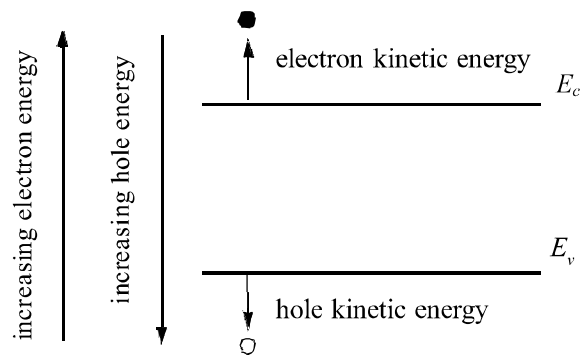
1.4 Semiconductors, Insulators, and Conductors



- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower E_g 's than insulators and can be doped.

Slide 1-10

1.5 Electrons and Holes



- Both electrons and holes tend to seek their lowest energy positions.
- Electrons tend to fall in the energy band diagram.
- Holes float up like bubbles in water.

Slide 1-11

1.5.1 Effective Mass

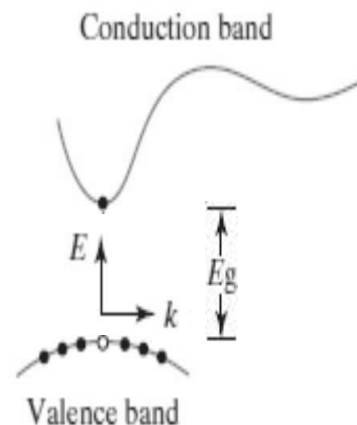
The electron wave function is the solution of the three dimensional Schrodinger wave equation

$$-\frac{\hbar^2}{2m_0} \nabla^2 \psi + V(r) \psi = E \psi$$

The solution is of the form $\exp(\pm \mathbf{k} \cdot \mathbf{r})$
 \mathbf{k} = wave vector = $2\pi/\text{electron wavelength}$
 For each \mathbf{k} , there is a corresponding E .

$$\text{acceleration} = -\frac{q\varepsilon}{\hbar^2} \frac{d^2 E}{dk^2} = \frac{F}{m}$$

$$\text{effective mass} \equiv \frac{\hbar^2}{d^2 E / dk^2}$$



Slide 1-12

1.5.1 Effective Mass

In an electric field, \mathbf{E} , an electron or a hole accelerates.

$$a = \frac{-q\mathcal{E}}{m_n} \quad \text{electrons}$$

$$a = \frac{q\mathcal{E}}{m_p} \quad \text{holes}$$

Electron and hole effective masses

	Si	Ge	GaAs	InAs	AlAs
m_n/m_0	0.26	0.12	0.068	0.023	2
m_p/m_0	0.39	0.3	0.5	0.3	0.3

Slide 1-13

1.5.2 How to Measure the Effective Mass

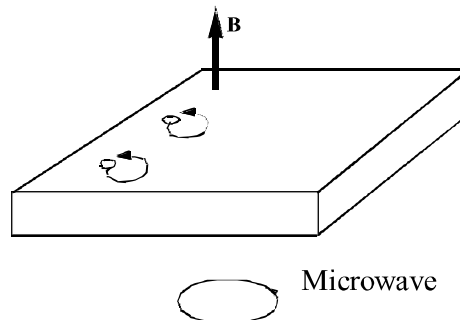
Cyclotron Resonance Technique

Centripetal force = Lorentzian force

$$\frac{m_n v^2}{r} = qvB$$

$$v = \frac{qBr}{m_n}$$

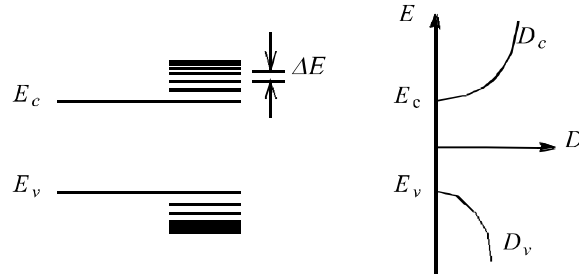
$$f_{cr} = \frac{v}{2\pi r} = \frac{qB}{2\pi m_n}$$



- f_{cr} is the Cyclotron resonance frequency.
- It is independent of v and r .
- Electrons strongly absorb microwaves of that frequency.
- By measuring f_{cr} , m_n can be found.

Slide 1-14

1.6 Density of States



$$D_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \left(\frac{1}{\text{eV} \cdot \text{cm}^3} \right)$$

$$D_c(E) \equiv \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3}$$

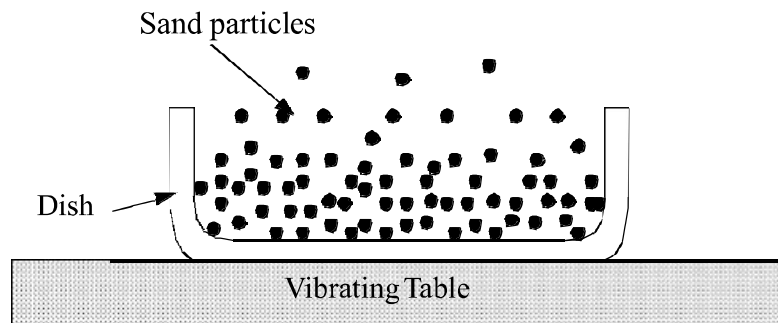
$$D_v(E) \equiv \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3}$$

Derived in Appendix I

Slide 1-15

1.7 Thermal Equilibrium and the Fermi Function

1.7.1 An Analogy for Thermal Equilibrium



- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy.

Slide 1-16

Appendix II. Probability of a State at E being Occupied

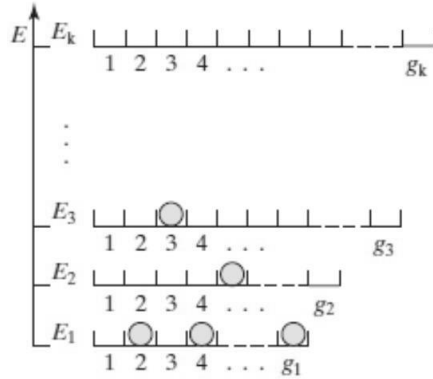
• There are g_1 states at E_1 , g_2 states at E_2 ... There are N electrons, which constantly shift among all the states but the average electron energy is fixed at $3kT/2$.

• There are many ways to distribute N among n_1, n_2, n_3, \dots and satisfy the $3kT/2$ condition.

• The equilibrium distribution is the distribution that maximizes the number of combinations of placing n_1 in g_1 slots, n_2 in g_2 slots.... :

$$n_i/g_i = \frac{1}{1 + e^{(E - E_F)/kT}}$$

E_F is a constant determined by the condition $\sum n_i = N$



1.7.2 Fermi Function—The Probability of an Energy State Being Occupied by an Electron

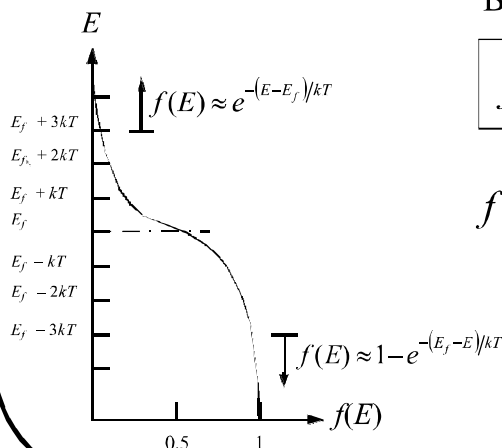
$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

E_f is called the *Fermi energy* or the *Fermi level*.

Boltzmann approximation:

$$f(E) \approx e^{-(E - E_f)/kT} \quad E - E_f \gg kT$$

$$f(E) \approx 1 - e^{-(E_f - E)/kT} \quad E - E_f \ll -kT$$



Remember: there is only one Fermi-level in a system at equilibrium.

1.8 Electron and Hole Concentrations

1.8.1 Derivation of n and p from $D(E)$ and $f(E)$

$$n = \int_{E_c}^{\text{top of conduction band}} f(E) D_c(E) dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c} e^{-(E - E_f)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{-(E_c - E_f)/kT} \int_0^{E - E_c} \sqrt{E - E_c} e^{-(E - E_c)/kT} d(E - E_c)$$

Slide 1-19

Electron and Hole Concentrations

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$N_c \equiv 2 \left[\frac{2\pi m_n kT}{h^2} \right]^{3/2}$$

N_c is called the *effective density of states (of the conduction band)*.

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$N_v \equiv 2 \left[\frac{2\pi m_p kT}{h^2} \right]^{3/2}$$

N_v is called the *effective density of states of the valence band*.

Remember: the closer E_f moves up to E_c , the larger n is; the closer E_f moves down to E_v , the larger p is.

For Si, $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$.

Slide 1-20

1.8.2 The Fermi Level and Carrier Concentrations

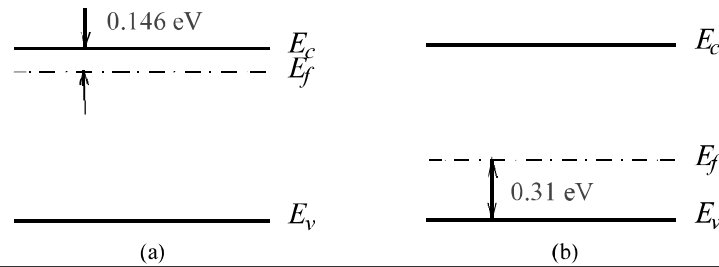
Where is E_f for $n = 10^{17} \text{ cm}^{-3}$? And for $p = 10^{14} \text{ cm}^{-3}$?

Solution: (a) $n = N_c e^{-(E_c - E_f)/kT}$

$$E_c - E_f = kT \ln(N_c/n) = 0.026 \ln(2.8 \times 10^{19} / 10^{17}) = 0.146 \text{ eV}$$

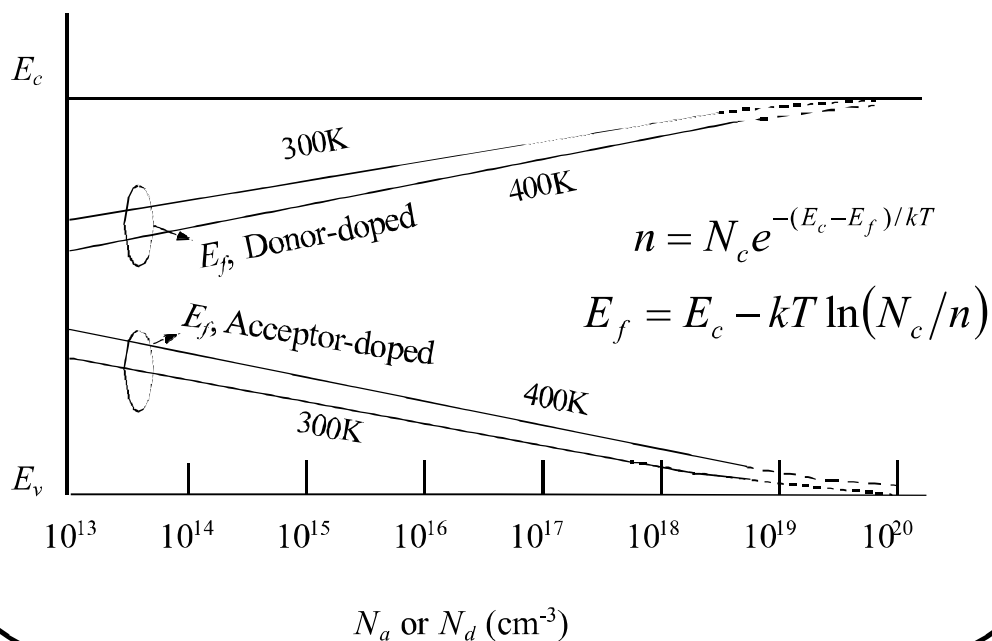
(b) For $p = 10^{14} \text{ cm}^{-3}$, from Eq.(1.8.8),

$$E_f - E_v = kT \ln(N_v/p) = 0.026 \ln(1.04 \times 10^{19} / 10^{14}) = 0.31 \text{ eV}$$



Slide 1-21

1.8.2 The Fermi Level and Carrier Concentrations



Slide 1-22

1.8.3 The np Product and the Intrinsic Carrier Concentration

Multiply $n = N_c e^{-(E_c - E_f)/kT}$ and $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$np = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

- In an intrinsic (undoped) semiconductor, $n = p = n_i$.
- n_i is the ***intrinsic carrier concentration***, $\sim 10^{10} \text{ cm}^{-3}$ for Si.

Slide 1-23

EXAMPLE: Carrier Concentrations

Question: What is the hole concentration in an N-type semiconductor with 10^{15} cm^{-3} of donors?

Solution: $n = 10^{15} \text{ cm}^{-3}$.

$$p = \frac{n_i^2}{n} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} = 10^5 \text{ cm}^{-3}$$

After increasing T by 60°C , n remains the same at 10^{15} cm^{-3} while p increases by about a factor of 2300 because $n_i^2 \propto e^{-E_g/kT}$.

Question: What is n if $p = 10^{17} \text{ cm}^{-3}$ in a P-type silicon wafer?

Solution:

$$n = \frac{n_i^2}{p} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}} = 10^3 \text{ cm}^{-3}$$

Slide 1-24

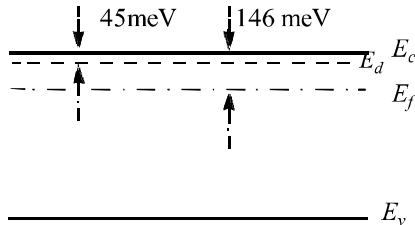
1.9 General Theory of n and p

EXAMPLE: Complete ionization of the dopant atoms

$N_d = 10^{17} \text{ cm}^{-3}$. What fraction of the donors are not ionized?

Solution: First assume that all the donors are ionized.

$$n = N_d = 10^{17} \text{ cm}^{-3} \Rightarrow E_f = E_c - 146 \text{ meV}$$



$$\text{Probability of not being ionized} \approx \frac{1}{1 + \frac{1}{2} e^{(E_d - E_f)/kT}} = \frac{1}{1 + \frac{1}{2} e^{((146-45)\text{meV})/26\text{meV}}} = 0.04$$

Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.

Slide 1-25

1.9 General Theory of n and p

$$\text{Charge neutrality: } n + N_a = p + N_d$$

$$np = n_i^2$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

Slide 1-26

1.9 General Theory of n and p

I. $N_d - N_a \gg n_i$ (i.e., N-type)

$$\begin{aligned} n &= N_d - N_a \\ p &= n_i^2 / n \end{aligned}$$

If $N_d \gg N_a$, $n = N_d$ and $p = n_i^2 / N_d$

II. $N_a - N_d \gg n_i$ (i.e., P-type)

$$\begin{aligned} p &= N_a - N_d \\ n &= n_i^2 / p \end{aligned}$$

If $N_a \gg N_d$, $p = N_a$ and $n = n_i^2 / N_a$

Slide 1-27

EXAMPLE: Dopant Compensation

What are n and p in Si with (a) $N_d = 6 \times 10^{16} \text{ cm}^{-3}$ and $N_a = 2 \times 10^{16} \text{ cm}^{-3}$ and (b) additional $6 \times 10^{16} \text{ cm}^{-3}$ of N_a ?

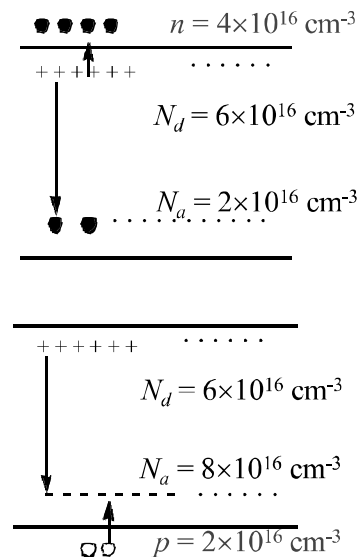
(a) $n = N_d - N_a = 4 \times 10^{16} \text{ cm}^{-3}$

$$p = n_i^2 / n = 10^{20} / 4 \times 10^{16} = 2.5 \times 10^3 \text{ cm}^{-3}$$

(b) $N_a = 2 \times 10^{16} + 6 \times 10^{16} = 8 \times 10^{16} \text{ cm}^{-3} > N_d$

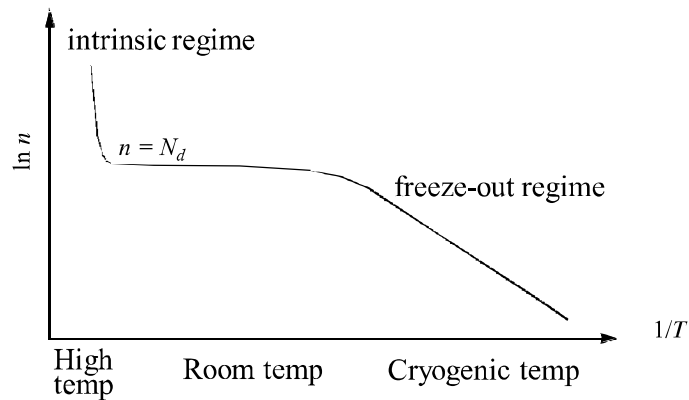
$$p = N_a - N_d = 8 \times 10^{16} - 6 \times 10^{16} = 2 \times 10^{16} \text{ cm}^{-3}$$

$$n = n_i^2 / p = 10^{20} / 2 \times 10^{16} = 5 \times 10^3 \text{ cm}^{-3}$$



Slide 1-28

1.10 Carrier Concentrations at Extremely High and Low Temperatures



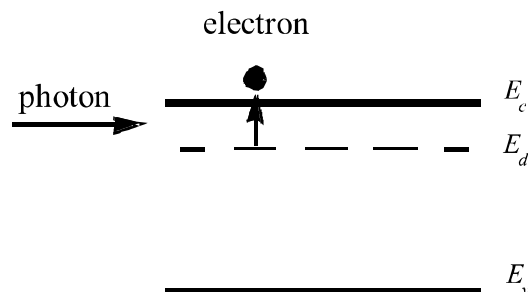
$$\text{high T: } n = p = n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

$$\text{low T: } n = \left[\frac{N_c N_d}{2} \right]^{1/2} e^{-(E_c - E_d)/2kT}$$

Slide 1-29

Infrared Detector Based on Freeze-out

- To image the black-body radiation emitted by tumors requires a photodetector that responds to $h\nu$'s around 0.1 eV.
- In doped Si operating in the freeze-out mode, conduction electrons are created when the infrared photons provide the energy to ionize the donor atoms.



Slide 1-30

1.11 Chapter Summary

Energy band diagram. Acceptor. Donor. m_n, m_p .
Fermi function. E_f .

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^2$$