Introduction To Topology

Q1: Let T be the collection of subsets of $\mathbb{N}$ consisting of empty set $\emptyset$ and all subsets of $\mathbb{N}$ of the form

$G_m = \{ m, m+1, m+2, \ldots \}, m \in \mathbb{N}$. Then Show that T is a topology on $\mathbb{N}$.

Q2: Let X be any set and T be the collection of all those subsets of X whose complements are finite together with empty set $\emptyset$. Then show that T is a topology on X.

Q3: Let X be a nonempty set and T be the collection of all sub sets of X. Prove that T is a topology on X.

Q4: Let X be any set and T be the collection of all those subsets of X whose complements are countable together with empty set $\emptyset$. Then show that T is a topology on X.

Q5: Let T be the collection of subsets of $\mathbb{N}$ consisting of empty set $\emptyset$ and all subsets of $\mathbb{N}$ of

the form $G_n = \{ 1,2,3, \ldots, n\}, n \in \mathbb{N}$. Then show that T is a topology on $\mathbb{N}$.

Q6: Let U consist of ϕ and all those subsets G of $\mathbb{R}$ having the property that to each $x \in G$, there exists $\varepsilon > 0$ such that $(x - \varepsilon, x + \varepsilon) \subseteq G$. Then prove that U is a topology on $\mathbb{R}$.

Q7: Give five different topologies on the set $X = \{a, b, c\}$.

Q8: (A) Let $X = \{ a, b, c\}$ and $T = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Are the following subsets of X, T −neighborhood of b? Explain your answer. (i) $\{a, b2\}$ (ii) $\{b, c\}$ (iii) X.

Q9: Prove or disprove that: If (X, T) is a topological space and $x \in X$, then the intersection of any two neighborhoods of x is a neighborhood of x.

Q10: Prove or disprove that: In every topological space(X, T), $D(A \cap B) = D(A) \cap D(B)$, for any sets A and B.

Q11: Prove or disprove that: Let (X, T) be a topological space and $A, B \subseteq X$, if $A \subseteq B$, then $\bar{A} \subseteq \bar{B}$.

Q12: Prove or disprove that: Let (X, T) be a topological space and $A, B \subseteq X$, then $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

Q13: Prove or disprove that: In every topological space (X,T), $\overline{A\cap B}=\bar{A}\cap\bar{B}$, for any sets A and B.

Q14: Let $X=\{1,2,3,4,5\}$, $T=\{\emptyset,\{2\},\{2,3\},\{3,4,5\},\{2,3,4,5\},\{1,2,3\},X\}$ and $B=\{\{2,3\}\}$. Then: (i) Find all neighbourhoods of 4. (ii) Write a topology on *X*, weaker than *T*. (iii) Write a topology on *X*, stronger than *T*. (iv) Is *T* a door space on *X*. (v) Is *B* a local base for 2 ? why?

Q15: Let $X=\{a,b,c,d,e\}$, and $T=\{\emptyset,\{a\},\{b,c\},\{a,b,c\},X\}$. Find whether $B=\{\{a,b,c\},X\}$ form a local base for c. Explain your answer?

Q16: Let $X=\{a,b,c,d\}$ and $T=\{\emptyset,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},\{b,c,d\},X\}$. Is the collection $\{\{a\},\{b\},\{c,d\}\}$ a base for T? Explain your answer.

Q17: If (X,D) be the discrete topological space, then show that the set $B=\{\{x\}:x\in X\}$ is a base for D.

Q18: Let $X=\{a,b,c,d,e\}$, and $T=\{\emptyset,\{a\},\{b,c\},\{a,b,c\},X\}$. Find whether $B=\{\{a,b\},X\}$ form a local base for a. Why?

Q19: Prove that the usual topological space $(\mathbb{R},U)$ is a (i) first countable space (ii) second countable space.

Q20: Show that the co-countable topology on an uncountable set X is not a Hausdorff space.

Q21: Show that every discrete space is a Hausdorff space.

Q22: Let $X=\{1,2,3\}$ and $T=\{\emptyset,\{1\},\{1,2\},\{1,3\},X\}$. If $A=\{2,3\}$ and $B=\{1,3\}$, then find the following: (i) All limit points of *A* and *B*. (ii) All isolated points of A and *B*.

Q23: Let $X=\{1,2,3,4\}$, and D be the discrete topology on X. If $S=\{2,4\}$, then Find all : (i) cluster points, (ii) isolated points, (iii) adherent points, of S.

Q24: Consider the usual topology $(\mathbb{R},U)$. Then find (i) interior (ii) exterior (iii) frontier (iv) derived (v) closure, for the sets $A=\mathbb{Q}$ (the set of rational numbers) and $B=(4,6]$.

Q25: Consider the usual topological space $(\mathbb{R},U)$, then find the closure of each of the following sets:

(i) $A=(0,1)$ (ii) $\mathbb{N}$ (iii) $\mathbb{Q}$ (iv) $B=\{\frac{1}{n}:n\in\mathbb{Z}^+\}$

Q26: Let $X = \{1,2,3,4,5\}$ and $T = \{\emptyset, \{2\}, \{4,5\}, \{2,4,5\}, \{1,3,4,5\}, X\}$. If $S = \{2,3,4\}$, then find the following: (i) $D(S)$ (ii) Adh(S) (iii) Isolated points of S.

Q27: Let $X = \{a, b, c, d\}$, and D be the discrete topology on X. If $S = \{a, b, c\}$, then find all:

(i) cluster points, (ii) isolated points, (iii) adherent points, of S.

Q28: Consider the usual topology $(\mathbb{R}, U)$. Then find (i) interior (ii) exterior (iii) frontier (iv) derived (v) closure, for the sets $A = \{\frac{1}{n}\ ; n \in \mathbb{Z}^+\}$, $B = \mathbb{Q}$ (the set of rational numbers) and $C = (0,2]$.

Q29: Let $X = \{1,2,3,4,5\}$, $T = \{\phi, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}, X\}$ and $Y = \{1,4,5\}$. Find the T- relative topology for Y.

Q30: Let $X = \{a, b, c, d\}$ and $T = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, X\}$. If $f: (X, T) \to (X, T)$ is a function defined by $f(a) = b,\ f(b) = d,\ f(c) = b,$ and $f(d) = c$, then find whether f is continuous at $x = a$ or not. Explain your answer.